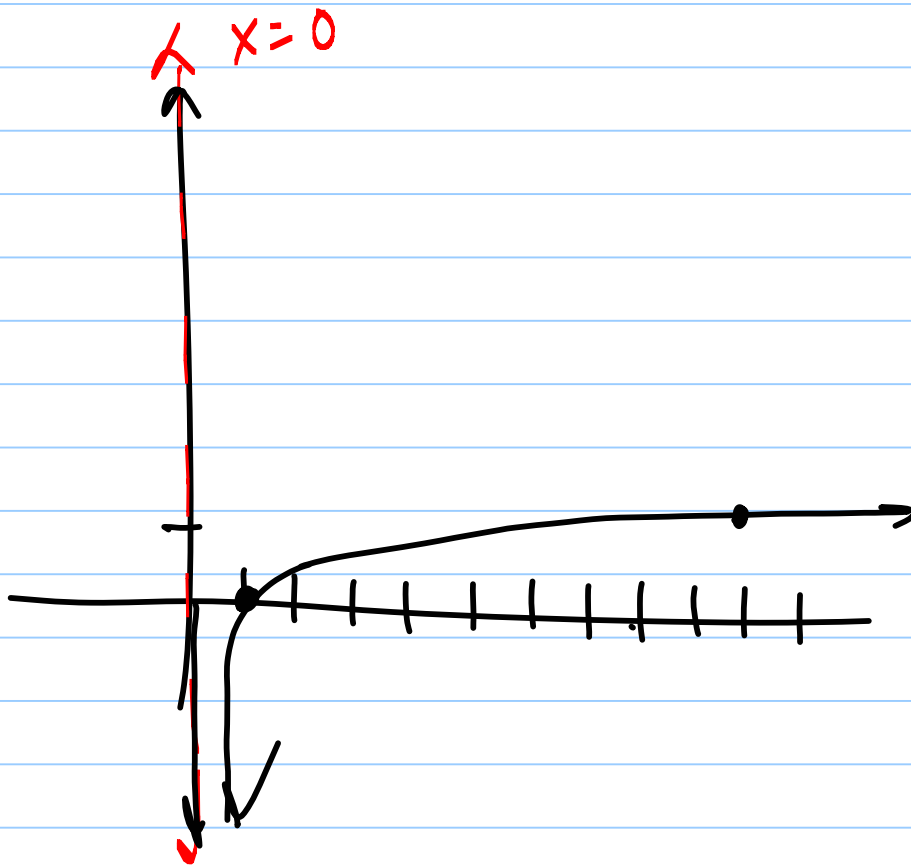


# Sections 3.3 & 3.5 / Properties of Log.

Note Title

7/8/2009



$$f(x) = \log_{10}(x)$$

Definition:  $\log_b x = y$  if and only if  $(*)$   
 $b^y = x.$

Base 10: "common logarithm" "log"

Base 'e': "natural logarithm" "ln"

⊛  $\ln(x) = y$  if and only if  $e^y = x$ .

$\log(x)$  common log (base 10)

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Ex) Rewrite  $\ln(x+y) = z$  in exponential form. (Assume  $x, y, z$  are positive)

Answer:  $e^z = x+y$

Ex) Rewrite  $10^q = m$  in logarithmic form.

Answer:  $\log(m) = q$

## Properties of Log / ln:

$$\textcircled{1} \log(ab) = \log(a) + \log(b)$$

$$\textcircled{2} \log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\textcircled{3} \log(a^n) = \underline{\underline{n \log(a)}}$$

$$\textcircled{4} \log 10^a = a \log(10) = a$$

Ex) Expand  $\log(xyz)$  (Assume  $x, y, z > 0$ )

$$= \log x + \log y + \log z$$

Ex) Condense:  $\ln a - 2 \ln b$   
(into a single logarithm)  $= \ln a - \ln b^2$   
 $= \ln \left( \frac{a}{b^2} \right)$

# Exponential Equations .

Solve:  $e^{8x} = 4$

"Take the natural logarithm of both sides"

$$\ln e^{8x} = \ln 4$$

$$8x \ln e = \ln 4$$

$$\frac{8x}{8} = \frac{\ln 4}{8}$$

$$x = \frac{(\ln 4)}{8}$$

$$8x = 1.386294...$$

$$x \approx .1733$$

$$\text{Ex] Solve: } \frac{3 \ln(x) - 4}{+4} = \frac{-2}{+4}$$

$$3 \ln(x) = 2$$

$$\ln_e(x) = \frac{2}{3}$$

Now, rewrite in exponential form

$$\boxed{e^{\frac{2}{3}} = x}$$

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$$\text{Ex] Solve: } 4^x + 9 = 12$$

$$4^x = 3$$

$$\log 4^x = \log 3 \quad \text{or} \quad \ln 4^x = \ln 3$$

$$\frac{x \log 4}{\log 4} = \frac{\log 3}{\log 4} \quad \text{or} \quad x \ln 4 = \ln 3$$

$$x = \frac{\log 3}{\log 4}$$

or

$$\frac{\ln 3}{\ln 4}$$

↖ ↗  
SAME!

$$x \approx .792\dots$$

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### Example from Online HW

Using  $\ln 2 \approx .693$

$$\ln(a) = 1.236$$

$$\ln 3 \approx 1.099$$

$$\ln(b) = 2.298$$

Compute:

a)  $\ln\left(\frac{3}{a^4}\right)$

b)  $\ln(12)$ .

$$\begin{aligned} \text{a) } \ln\left(\frac{3}{a^4}\right) &= \ln 3 - \ln a^4 \\ &= \underline{\ln 3} - 4 \ln a \\ &= 1.099 - 4(1.236) \\ &= \boxed{-3.845} \end{aligned}$$

$$\begin{aligned} \text{b) } \ln(\underline{\underline{12}}) &= \ln(2 \cdot 2 \cdot 3) \\ &= \ln 2 + \ln 2 + \ln 3 \\ &= .693 + .693 + 1.099 \end{aligned}$$