Section 5.3/5.4 MATRIX ARITHMETIC & INVERSES

Note Title 7/23/2009

A matrix is a rectangular array of numbers.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} -2 & 3 & 5 \\ 1 & 0 & -9 \end{bmatrix}_{2\times 3}$$

① Add/Subtract Matrices → Matrices must be same size.

$$A+B = \begin{bmatrix} 0 & 7 & 11 \\ 2 & 2 & -6 \end{bmatrix} \qquad A-B = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & 12 \end{bmatrix}$$

② Multiplying a matrix by a constant, and the distributive property hold:

$$k[A+B] = kA + kB$$

$$3(A+B) = \begin{bmatrix} 0 & 21 & 33 \\ 6 & 6 & -18 \end{bmatrix} = 3A + 3B$$

MATRIX MULTIPLICATION: Need to check a few things.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$$

🟪 × 🟧

↑ ↑

# of rows # of columns

🟧3 × 🟪2

↑ ↑

# of rows # of columns

🟧 : Inner dimensions must match

🟪 : AB will be a 2×2 matrix

Can you compute BA? Yes: Answer is a 3×3.

* Note $AB \neq BA$; matrix multiplication not commutative in general.

Ex] $\begin{bmatrix}5 & 7 & 9\end{bmatrix}_{1\times3} \begin{bmatrix}2\\3\\5\end{bmatrix}_{3\times1}$ Answer: 1x1

$5\cdot2 + 7\cdot3 + 9\cdot5 = [76]$

Ex] $\begin{bmatrix}\text{[redacted]}\end{bmatrix}_{1\times3} \begin{bmatrix}2 & \text{[redacted]}\\3 & \\5 & \end{bmatrix}_{3\times2}$ Answer: 1 x 2

Answer: $\begin{bmatrix}76 & 16\end{bmatrix}$

$5\cdot1 + 7\cdot-1 + 9\cdot2$
$= 5 - 7 + 18$
$= 16$

Ex)

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

2x2 2x1

Answer: 2x1

$$\begin{bmatrix} 2 \cdot -1 + 1 \cdot 5 \\ 3 \cdot -1 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Answer

Ex)

$$\begin{bmatrix} \text{[illegible]} & \text{[illegible]} \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix}$$

Answer: 2x2

$$\begin{bmatrix} 3 & 2 \cdot 3 + 1 \cdot -2 \\ 2 & 3 \cdot 3 + 1 \cdot -2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 3 & 4 \\ 2 & 7 \end{bmatrix}}$$

Answer

Identity Matrix: $n \times n$ matrix

(same # of rows as columns)

1's on the diagonal

from left to right

0's everywhere else.

2x2 Identity: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3x3 Identity: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Inverse of Matrix ; $(2\overset{a}{\times}2$ matrix)

$$\underbrace{\begin{bmatrix} a & c \\ b & d \end{bmatrix}}_{A}\underset{\uparrow}{\begin{bmatrix} \quad ? \quad \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse of A

A^{-1}

Formula: To find A^{-1}, when $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$,

★ $$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

Ex) Find the inverse of $A = \begin{bmatrix} 2 & 0 \\ 5 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 5 & 0 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1/5 \end{bmatrix}$$

So....

$$\begin{bmatrix} 1/2 & 0 \\ -1/2 & 1/5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Matrix Equations:

$$2x + 3y = -4$$

$$x + 2y = -3$$

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}}_{\text{if Invertible}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

Matrix Equation

$$\underbrace{A^{-1}A} \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\text{Identity} \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix} = \begin{bmatrix} \# \\ \# \end{bmatrix}$$