

# Section 5.3/5.4 MATRIX ARITHMETIC

Note Title

7/23/2009

## & INVERSES

A matrix is a rectangular array of numbers.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}_{2 \times 3}$$

$$B = \begin{bmatrix} -2 & 3 & 5 \\ 1 & 0 & -9 \end{bmatrix}_{2 \times 3}$$

① Add/Subtract Matrices  $\rightarrow$  Matrices must be same size.

$$A + B = \begin{bmatrix} 0 & 7 & 11 \\ 2 & 2 & -6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & 12 \end{bmatrix}$$

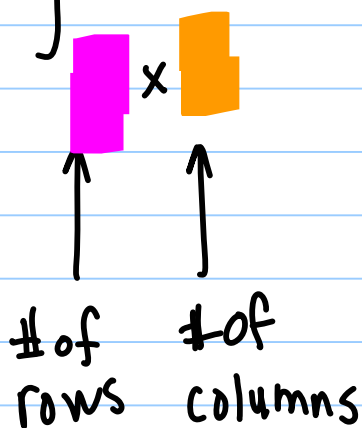
② Multiplying a matrix by a constant, and the distributive property hold:

$$k[A + B] = kA + kB$$

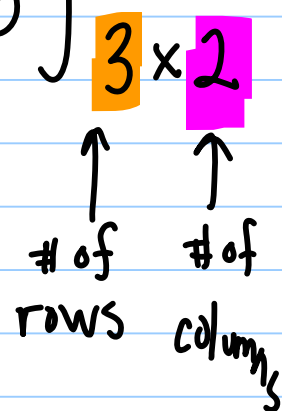
$$3(A + B) = \begin{bmatrix} 0 & 21 & 33 \\ 6 & 6 & -18 \end{bmatrix} = 3A + 3B$$

MATRIX MULTIPLICATION: Need to check  
a few things.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

  
# of rows   # of columns

$$B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$$

  
# of rows   # of columns

 : Inner dimensions must match

 :  $AB$  will be a  $2 \times 2$  matrix

Can you compute  $BA$ ? Yes: Answer is  
a  $3 \times 3$ .

\* Note  $AB \neq BA$ ; matrix multiplication  
not commutative in general.

$$\text{Ex)} \quad \begin{matrix} [5 & 7 & 9] \\ & & 1 \times 3 \end{matrix} \quad \begin{matrix} \left[ \begin{matrix} 2 \\ 3 \\ 5 \end{matrix} \right] \\ & & 3 \times 1 \end{matrix}$$

Answer:  $1 \times 1$

$$5 \cdot 2 + 7 \cdot 3 + 9 \cdot 5 = [76]$$

$$\text{Ex)} \quad \begin{matrix} [ \text{orange} ] \\ & & 1 \times 3 \end{matrix} \quad \begin{matrix} \left[ \begin{matrix} 2 & \text{orange} \\ 3 & \text{orange} \\ 5 & \text{orange} \end{matrix} \right] \\ & & 3 \times 2 \end{matrix}$$

Answer:

$1 \times 2$

Answer:  $\begin{bmatrix} 76 & 16 \end{bmatrix}$

$$\begin{aligned} & 5 \cdot 1 + 7 \cdot (-1) + 9 \cdot 2 \\ & = 5 - 7 + 18 \\ & = 16 \end{aligned}$$

Ex)  $\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix}$  Answer: 2x1

$2 \times 2 \quad 2 \times 1$

$$\begin{bmatrix} 2 \cdot -1 + 1 \cdot 5 \\ 3 \cdot -1 + 1 \cdot 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \underline{\underline{\text{Answer}}}$$

Ex)  $\begin{bmatrix} \phantom{2} & \phantom{1} \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 5 & -2 \end{bmatrix}$  Answer: 2x2

$$\begin{bmatrix} 3 & 2 \cdot 3 + 1 \cdot -2 \\ 2 & 3 \cdot 3 + 1 \cdot -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 2 & 7 \end{bmatrix} \underline{\underline{\text{Answer}}}$$

Identity Matrix :  $n \times n$  matrix

(same # of rows as columns)

1's on the diagonal  
from left to right

0's everywhere else.

$$2 \times 2 \text{ Identity: } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3 \times 3 \text{ Identity: } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

## Inverse of Matrix ; ( $2 \times 2$ matrix)

$$\underbrace{\begin{bmatrix} a & c \\ b & d \end{bmatrix}}_A \begin{bmatrix} ? \\ ? \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

↑  
Inverse of A  
 $A^{-1}$

Formula: To find  $A^{-1}$ , when  $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ ,

~~Ex~~  $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

Ex) Find the inverse of  $A = \begin{bmatrix} 2 & 0 \\ 5 & 5 \end{bmatrix}$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 5 & 0 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{5} \end{bmatrix}$$

$$\text{So } \dots \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Matrix Equations:

$$2x + 3y = -4$$

$$x + 2y = -3$$

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}}_{\text{if invertible}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

Matrix Equation

$$\underbrace{A^{-1}A}_{\text{Identity}} \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\text{Identity} \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} -4 \\ -3 \end{bmatrix} = \begin{bmatrix} \# \\ \# \end{bmatrix}$$

