4.5/4.6 Rational Functions

Note Title 7/20/2009

Rational Functions are of the form

$$r(x) = \frac{p(x)}{q(x)}, \quad q(x) \neq 0$$

Ex] $r(x) = \frac{1}{x}$

Features of Rational Functions

- Zeros
- Vertical Asymptotes → "Poles"
- Horizontal Asymptotes
- y-intercept

① Finding zeros: Set numerator equal to 0; Solve.

Ex) $f(z) = \dfrac{z-1}{z^2-9}$

$z - 1 = 0$

$\boxed{z = 1}$

Ex) $f(z) = \dfrac{z^3-1}{z^2-16}$

Hint: $z=1$ is a zero.

$z^3 - 1 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$

$$z^2 + z + 1 \longrightarrow$$

Use Quad. Formula:

$$\frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$$

$$\frac{-1}{2} \pm \frac{\sqrt{3}\,i}{2}$$

$$\boxed{\begin{aligned} z &= \frac{-1}{2} + \frac{\sqrt{3}\,i}{2} \\ z &= -\frac{1}{2} - \frac{\sqrt{3}\,i}{2} \end{aligned}}$$

(2) Finding Poles: Set denominator $= 0$; Solve,

$$f(x) = \frac{x^2 - 1}{x^2 - 9}$$

$$x^2 - 9 = 0$$

$$(x-3)(x+3) = 0$$

$$\boxed{x = \pm 3}$$

③ Horizontal Asymptote: (What value does the output approach)

$$r(x) = \frac{1}{x}$$

If $x \to \infty$, $r(x) \to 0$

$$r(x) = \frac{x^3 + 5x + 1}{\boxed{x^7} + x + 2}$$

[If degree numerator is smaller

$\boxed{y=0}$ is the horizontal asymptote

CASE 1

CASE 2: Degree in numerator & denominator are equal, then

$$\boxed{y = \frac{\text{Lead Coeff. numerator}}{\text{Lead coeff denominator}}}$$

Ex) $r(x) = \dfrac{4x^2 + 2x - 1}{\boxed{3x^2} + 5}$

$\dfrac{4\cancel{x^2}}{3\cancel{x^2}}$ $\boxed{y = \dfrac{4}{3}}$

Case 3: Degree in numerator greater by 1

$\dfrac{x^3}{x^2}$ for instance, "SLANT ASYMPTOTE"

Ex) Online HW

$$\frac{3x-15}{6x+19} = \frac{x-1}{2x+5}$$

$$\underbrace{(3x-15)(2x+5)} = \underbrace{(x-1)(6x+19)}$$

Multiply out →

$$\cancel{6x^2} + 15x - 30x - 75 = \cancel{6x^2} + 19x - 6x - 19$$

$$-15x - 75 = 13x - 19$$

$$-28x = 56$$

$$\boxed{x = -2}$$

Check answers.

$$\begin{array}{r} 75 \\ -19 \\ \hline 56 \end{array}$$

Section 4.6 : Polynomial and Rational Inequalities

#35 pg 437: # of employees in a startup company is given by $f(t) = \dfrac{30+40t}{5+2t}$

(t is # of months after company was organized)

(a) For what values of t is $f(t) < 18$?

$$\frac{30+40t}{5+2t} < 18$$

$$\frac{30+40t}{5+2t} - \frac{18}{1} < 0$$

$$\frac{30+40t-18(5+2t)}{(5+2t)} < 0$$

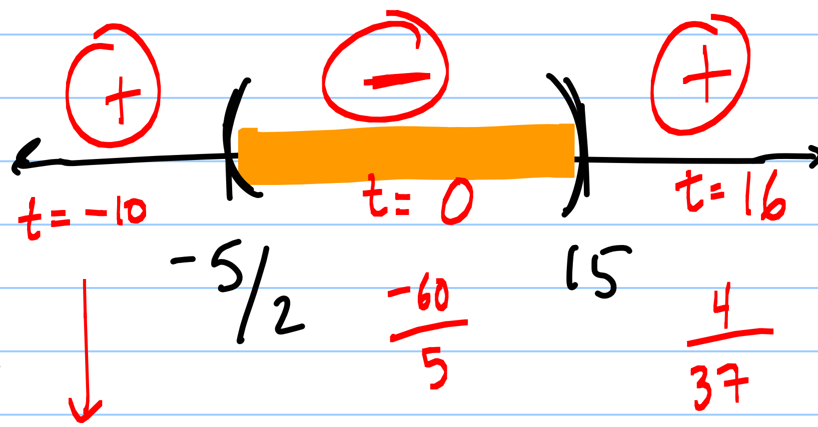
$$\frac{30+40t-90-36t}{5+2t} < 0$$

$$\boxed{\frac{4t-60}{5+2t} < 0}$$

Find zeros: $t = 15$

Poles: $t = -5/2$

Mark off on a # line



$$\frac{-40-60}{-15} = \frac{-100}{-15}$$

If this problem had no real life scenario associated to it, $\boxed{-\frac{5}{2} < t < 15}$ Answer.

$$\boxed{0 < t < 15}$$

For our word problem.

Ex] Solve: $x^3 + 6x^2 + 5x > 0$.

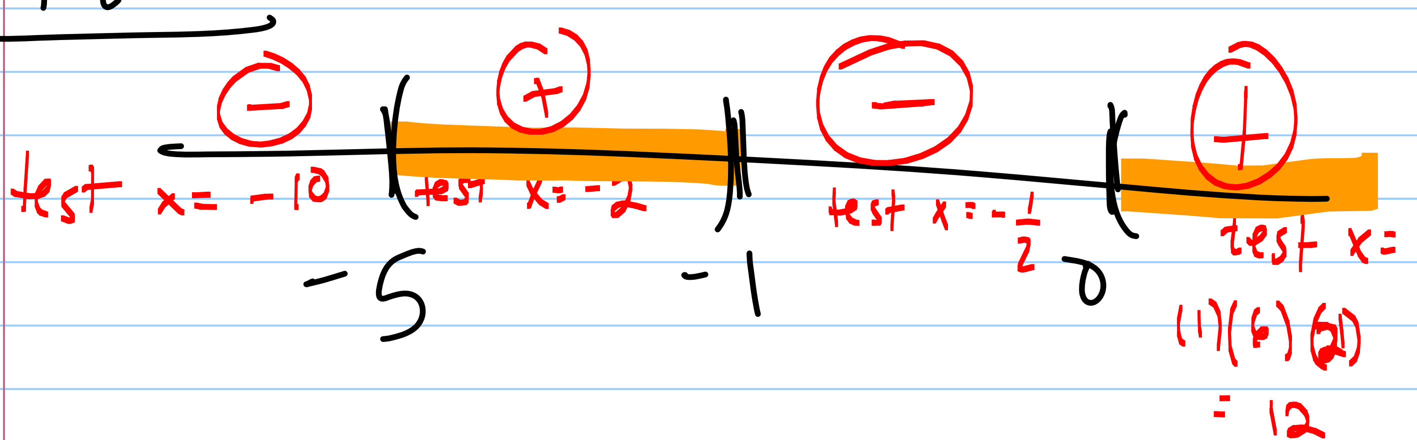
→ Have zero on right side.

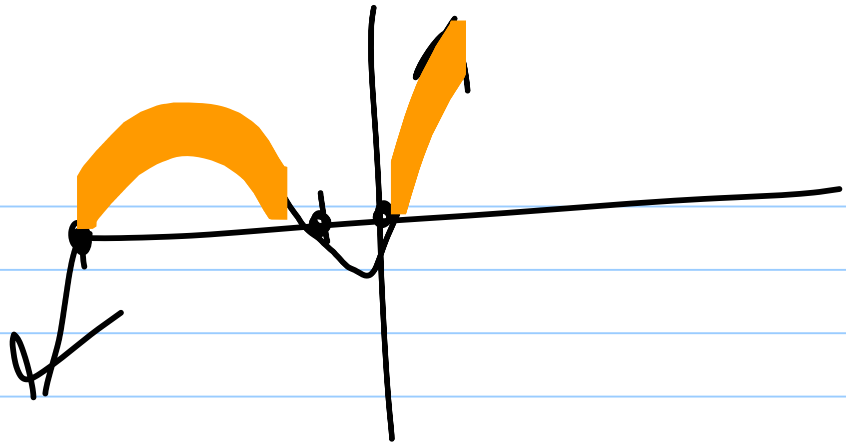
→ Factor left side

$$x(x^2 + 6x + 5) > 0$$

$$x(x+5)(x+1) > 0$$

Plot zeros on a number line





$$\boxed{\underline{\text{Answer:}}\quad x > 0 \text{ or } -5 < x < -1}$$