Section 4.4/ Roots of Polynomials

Note Title 7/15/2009

$f(x) = x^2 + 5x + 6$ Degree 2 polynomial

"Find the roots of $f(x)$"
"Find the zeros of $f(x)$"

$$x^2 + 5x + 6 = 0$$
$$(x+3)(x+2) = 0$$

$x = -3$
$x = -2$

"$x+3$ is a linear factor of $f(x)$"
"$x+2$ is a linear factor of $f(x)$"

Ex) Is $x+2$ a factor of $x^3+8x^2+21x+18$?

→ Plug in $x=-2$ and check if the output is 0.

$$(-2)^3+8(-2)^2+21(-2)+18$$

$$= -8+32-42+18$$

$$= 24-42+18 = \boxed{0}$$

Yes, $(x+2)$ is a factor.

Ex) Answer the above question with LONG DIVISION.

$$\begin{array}{r} x^2+6x+9 \\ (x+2)\overline{\smash{)}\,x^3+8x^2+21x+18} \\ -(x^3+2x^2) \\ \hline 6x^2+21x \\ -(6x^2+12x) \\ \hline 9x+18 \end{array}$$

$x^2(x+2)$
$= x^3+2x^2$

$6x(x+2)$
$= 6x^2+12x$

$9(x+2)$

Quotient: x^2+6x+9

$$-(9x+18)$$
$$\boxed{0}$$

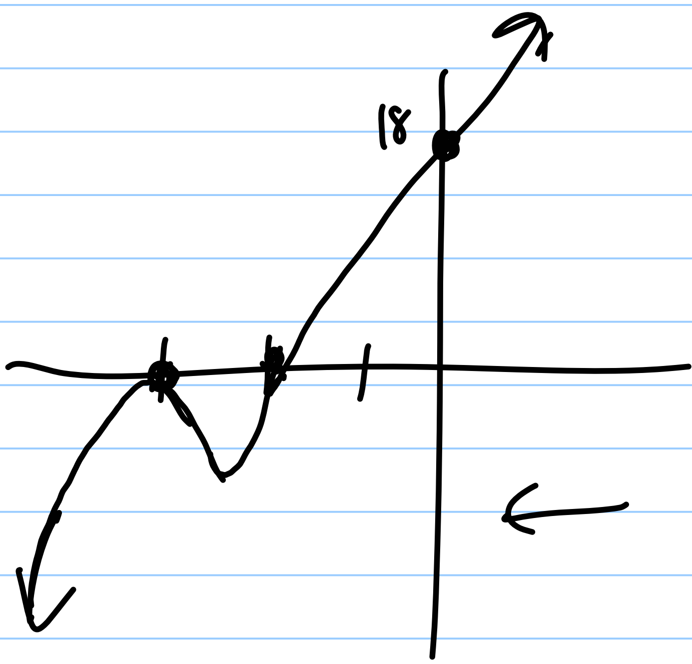
Since Remainder $=0$, $x+2$ is a factor of $\underline{\underline{x^3+8x^2+21x+18}}$

Ex) Write $x^3+8x^2+21x+18$ as a product of all its linear factors.

$$(x+2)(x^2+6x+9)$$
$$=(x+2)(x+3)(x+3)$$

$x=-2$ is a root.

$x=-3$ is a $\underline{\text{double}}$ root;



Graph of $f(x)=x^3+8x^2+21x+18$

Ex] Find a polynomial with zeros at $x = -2$, $x = 0$, and $x = 4$.

$$p(x) = x(x+2)(x-4)$$

$$p(x) = x(x^2 - 2x - 8)$$

$$\boxed{p(x) = x^3 - 2x^2 - 8x}$$ ← One possibility (online hw)

$$k(x^3 - 2x^2 - 8x)$$ would also work.

$k \neq 0$

$a+bi$

Ex | Polynomials with Complex Zeros.

Recall: $\sqrt{-1} = i$, where $i^2 = -1$ (by definition)

So, for instance

$$\sqrt{-9} = 3i \qquad \sqrt{-3} = \sqrt{3}\,i$$

$$\sqrt{-25} = 5i$$

Find all roots, both real & complex of $p(x) = x^3 + 2x^2 - 3$, given that $\underline{\underline{x=1}}$ is a root.

$$\begin{array}{c|cccc}
1 & 1 & 2 & 0 & -3 \\
 & \downarrow & 1 & 3 & -3 \\
\hline
 & 1 & 3 & 3 & \boxed{0}
\end{array}$$

coeff of x^2, x, constant

What else is a factor of $p(x)$?

$$x^2 + 3x + 3$$

$$p(x) = (x-1)\underbrace{(x^2+3x+3)}_{\text{Find zeros of this quadratic.}}$$

$$\frac{-3 \pm \sqrt{9-4(1)(3)}}{2(1)}$$

$$\frac{-3 \pm \sqrt{-3}}{2}$$

$$\frac{-3}{2} \pm \frac{\sqrt{3}\,i}{2}$$

* This polynomial has 1 real root & 2 complex roots.

* Complex roots occur in pairs.

$x = 1$

or

$-\frac{3}{2} + \frac{\sqrt{3}}{2}i$

or $-\frac{3}{2} - \frac{\sqrt{3}}{2}i$

WORD PROBLEM: The revenue from the sale of a product is given by $R(x) = -x^3 - 81x^2 + 1810x$. If 9 units brings in \$9000, find another # of units that brings in \$9000 in revenue.

$$\begin{array}{r|rrrr} 9 & -1 & -81 & 1810 & -9000 \\ & \downarrow & -9 & -810 & 9000 \\ \hline & -1 & -90 & 1000 & \boxed{0} \end{array}$$

$-x^2 - 90x + 1000$

$-1(x^2 + 90x - 1000)$

$-1(x+100)(x-10)$

$\boxed{x = 10 \text{ units}}$

~~$x = -100$~~ units