Section 4.1 & 4.3 / Polynomials

Note Title 7/13/2009

A polynomial is of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

(here, n is an integer ≥ 0)

Ex) $p(x) = 3x^4 - 5x^3 + 2x^2 - x + 3$

n: (degree, highest power)

$p(x)$ is of degree 4.

In this example $\begin{cases} a_4 = 3, \quad a_3 = -5, \quad a_2 = 2, \quad a_1 = -1 \\ a_0 = 3 \end{cases}$

a_n: Leading Coefficient

In $p(x)$, the leading coefficient is 4.

Ex] $h(x) = 2 + 3x - 5x^2 + 9x^7$

degree? 7

Leading Coefficient? 9

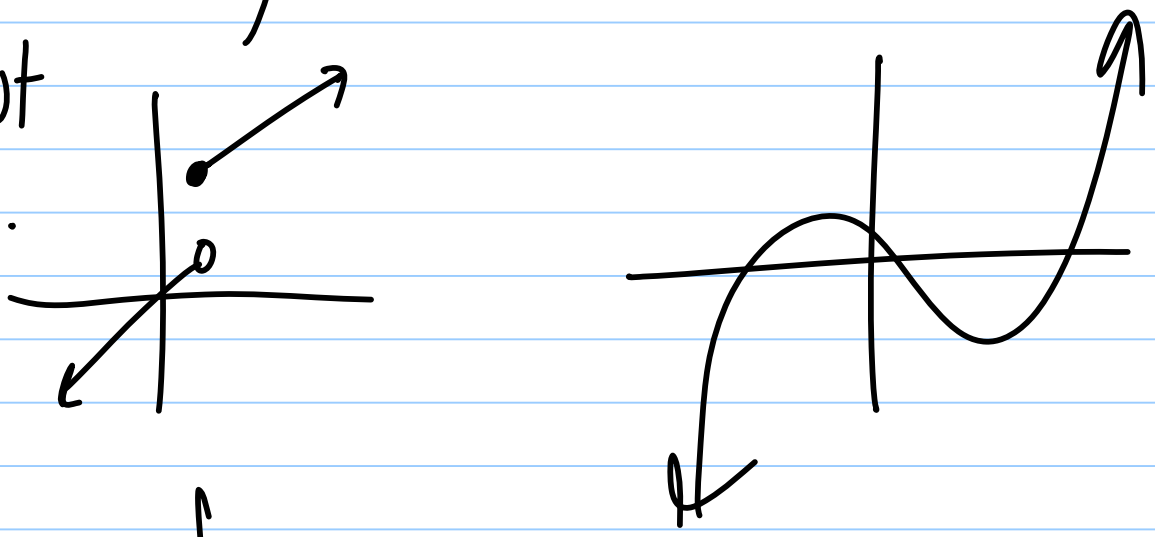
y-intercept? Set $x=0$, $\boxed{2}$

(constant term)

Features of Polynomials

- Domain is all $\mathbb{R}$'s.
- Polynomials are continuous

not cts.



- No sharp edges/corners.

* By examining the degree & leading coefficient of a polynomial, we can determine "where" the graph of the polynomial is rising and falling

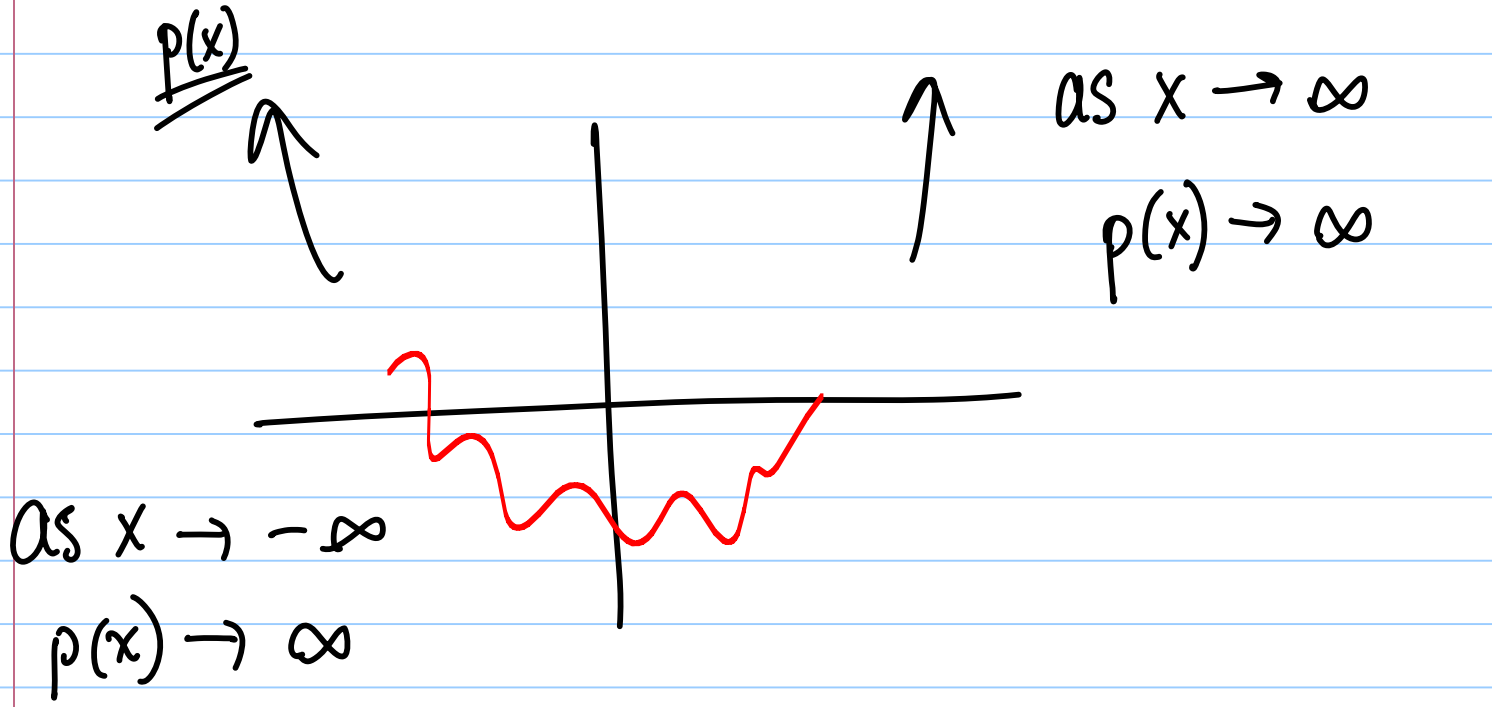
"Leading Term Test" (Analyzing end behavior)

As $x \to \infty$ (as x gets large, where does $p(x)$ head?)

As $x \to -\infty$ (as x gets very negative, where does $p(x)$ do?)

Case 1: n: even (degree even)

a_n: Positive (Leading coefficient)



Case 2: n: even (degree)

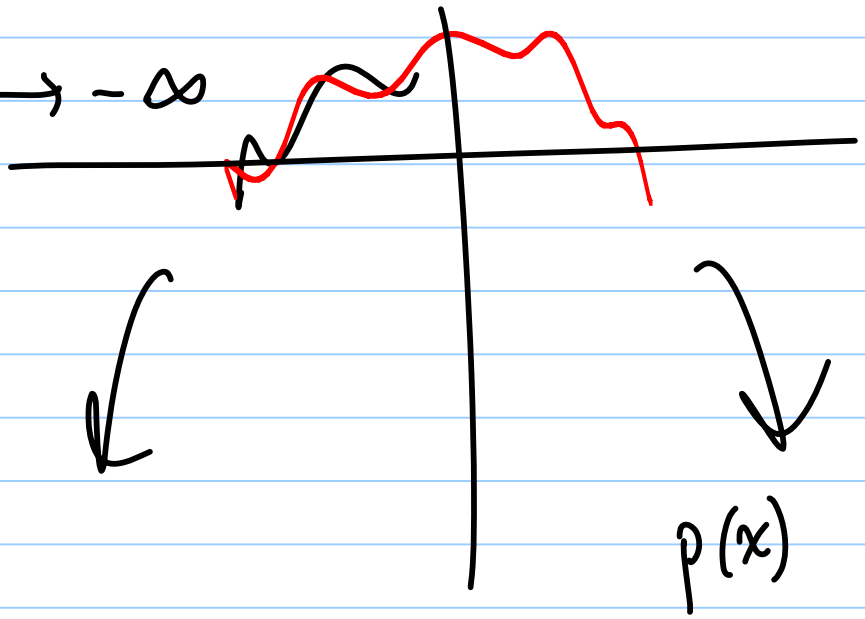
a_n: negative (lead. coefficient)

as $x \to -\infty$

$p(x) \to -\infty$

as $x \to \infty$

$p(x) \to -\infty$

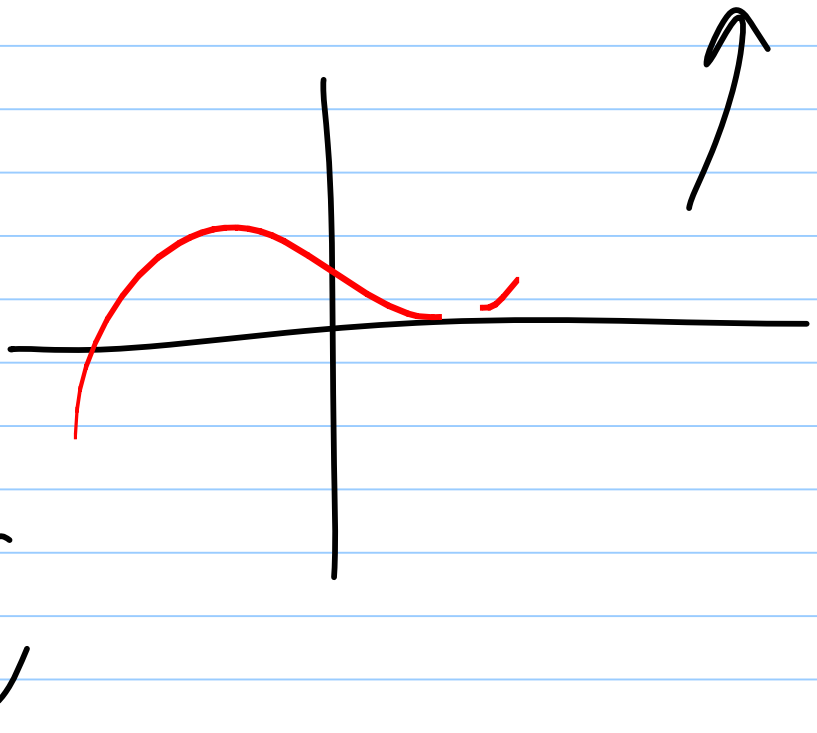


Case 3: n: odd degree

a_n: positive leading coefficient

as $x \to -\infty$

$p(x) \to -\infty$



as $x \to \infty$

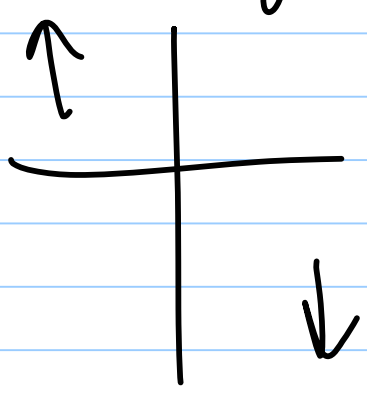
$p(x) \to \infty$

Case 4: n: odd degree

a_n: negative leading coeff.

as $x \to -\infty$

$p(x) \to \infty$



as $x \to \infty$

$p(x) \to -\infty$

- Polynomials of degree n have at most n x-intercepts (on our real x-y plane)

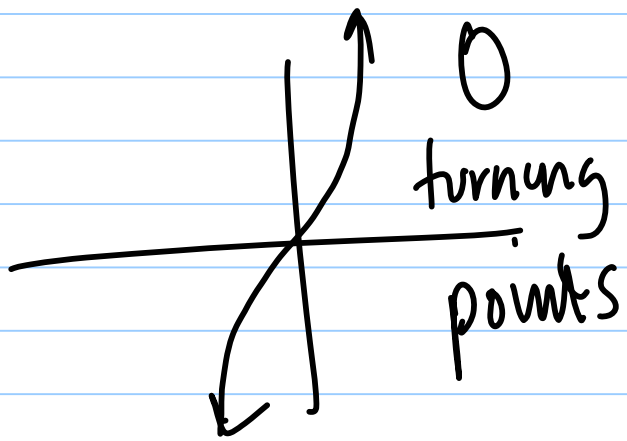
- Polynomials of degree n have <u>at most</u> n-1 turning points

Ex) $p(x) = (x)(x^2-1) = x^3 - x$ degree 3

2 turning points

# of turning points drop in pairs.

Ex) $p(x) = x^3$



0 turning points

Terminology: "Zeros"

"Roots" of a polynomial

[where the polynomial $p(x)=0$]

Ex] $p(x) = x^3 - x$. Find zeros. (roots)

Solve. $x^3 - x = 0$

$x(x^2 - 1) = 0$

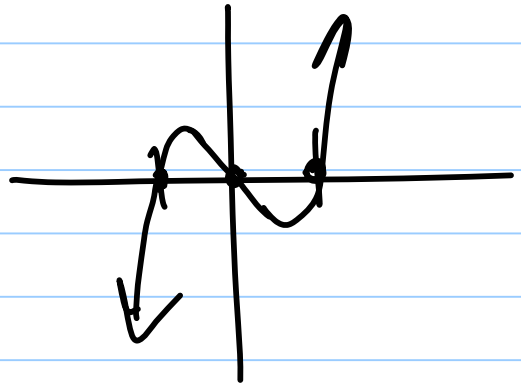
$(x)(x+1)(x-1) = 0$

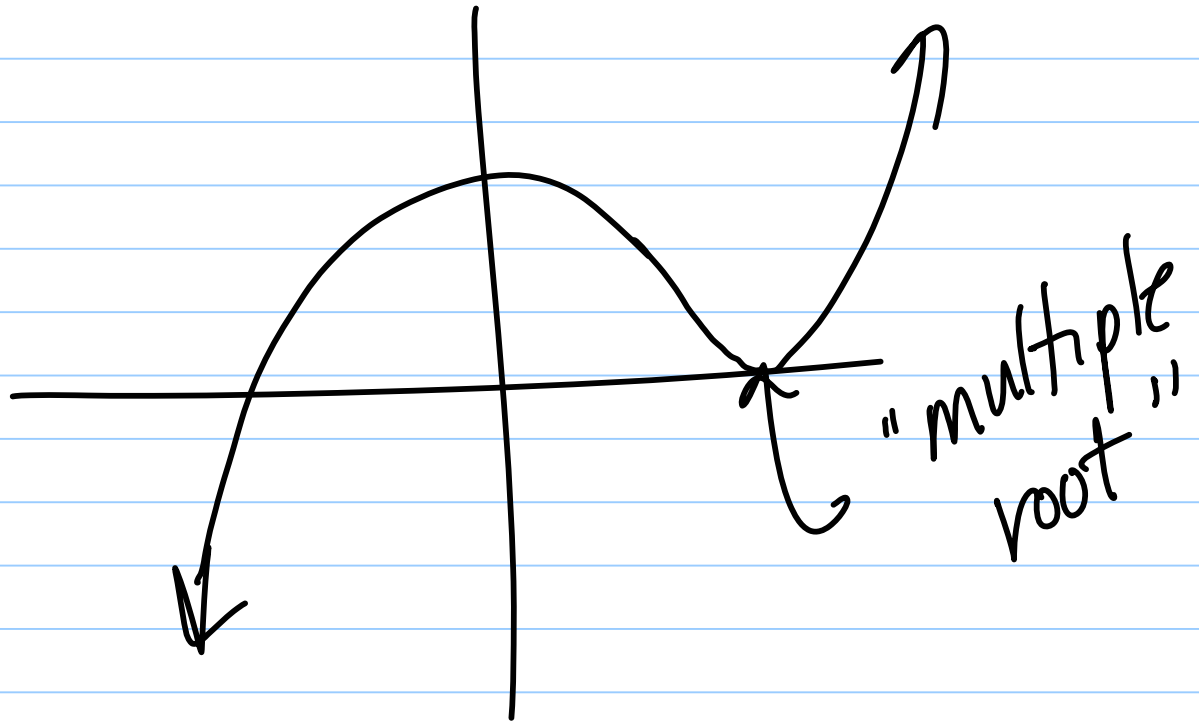
$x = 0$

$x = -1$

$x = 1$

zeros





$$p(x) = (x+3)(x-2)^2$$