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Math 100 Section E/F – Exam 3
April 10, 2007

Can also do these problems with ln for full credit

1. Rewrite the formula $y = \frac{8}{x^2}$ taking the logarithm of both sides (you may assume $x > 0$).

$$\log(y) = \log\left(\frac{8}{x^2}\right)$$
$$\log(y) = \log(8) - \log(x^2)$$
$$\boxed{\log(y) = \log(8) - 2\log(x)}$$
$$\log(y) = .9031 - 2\log(x)$$

2. Rewrite the formula $y = (1.3)7^x$ taking the logarithm of both sides.

$$\log(y) = \log((1.3)7^x)$$
$$\log(y) = \log(1.3) + \log(7^x)$$
$$\boxed{\log(y) = \log(1.3) + x\log(7)}$$
$$\log(y) = 0.1139 + 0.8451x$$

3. If $\log(a) = 1.2$ and $\log(b) = -3.1$, what is $\log\left(\frac{\sqrt{a}}{b^3}\right)$?

$$\log\left(\frac{\sqrt{a}}{b^3}\right) = \log(\sqrt{a}) - \log(b^3)$$
$$= \log(a^{1/2}) - \log(b^3)$$
$$= \frac{1}{2}\log(a) - 3\log(b)$$
$$= \frac{1}{2}(1.2) - 3(-3.1) = 0.6 + 9.3 = \boxed{9.9}$$

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4. Solve $e^{3x+2} = 1$.

$$\ln\left(e^{3x+2}\right) = \ln(1)$$

$$(3x+2)\cancel{\ln}(e)^{\nearrow 1} = \cancel{\ln(1)}^{\nearrow 0}$$

$$(3x+2)\cdot(1) = 0$$

$$3x+2 = 0$$

$$3x = -2$$

$$\boxed{x = -2/3}$$

5. Solve 3log(2x - 10) + 6 = 9.

$$3\log(2x-10) = 3$$

$$\log(2x-10) = 1$$

$$10^{\log(2x-10)} = 10^1$$

$$2x-10 = 10$$

$$2x = 20$$

$$\boxed{x = 10}$$

6. What is the domain of the function $f(x) = \log(6+4x)+1$?

Domain of $\log(u)$ is $u > 0$

So $6+4x > 0$

$$4x > -6$$

$$\boxed{x > -\frac{6}{4} = -3/2}$$

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7. What is the future value in 6 years of an initial investment of $100,000 at an annual interest rate of 6% compounded twice per year?

$$\left(1+\frac{.06}{2}\right)^{2\cdot 6} 100{,}000 = F.V.$$

$$(1.03)^{12}\cdot 100{,}000 = F.V.$$

$$\boxed{\$142{,}576.09 = F.V.}$$

8. A company's sales decay after an advertising campaign according to the model $S = 50{,}000e^{-0.08x}$, where x is the number of weeks after the campaign ended. How many weeks after the campaign ends will it take for sales to fall to $22,500?

$$50{,}000\, e^{-0.08x} = 22{,}500$$

$$e^{-0.08x} = \frac{22{,}500}{50{,}000} = 0.45$$

$$\ln(e^{-0.08x}) = \ln(0.45)$$

$$-0.08x \cancel{\ln(e)}^{1} = \ln(0.45)$$

$$-0.08x = -.7985....$$

$$x = \frac{-.7985...}{-0.08} = 9.98....$$

$$\boxed{10 \text{ weeks}}$$

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9. The magnitude of an earthquake is expressed on the Richter scale. The Richter scale is $R = \log\left(\frac{I}{I_0}\right)$, where I_0 is a minimal intensity and I is the measured intensity.

(a) If an earthquake has an intensity of $10{,}000I_0$ what is its magnitude?

$$R = \log\left(\frac{10{,}000 \cancel{I_0}}{\cancel{I_0}}\right) = \log(10000) = \log(10^4) = \boxed{4}$$

(b) How much does the magnitude increase if the intensity is 10 times larger than the intensity from part (a)?

$$10 \times 10{,}000 I_0 = 100{,}000 I_0$$

$$R = \log\left(\frac{100{,}000 \cancel{I_0}}{\cancel{I_0}}\right) = \log(100{,}000) = \log(10^5) = \underline{5}$$

$\boxed{\text{So magnitude increases by 1 (from 4 to 5)}}$

10. Find all real and complex solutions to $x^3 + x^2 - 4x - 24 = 0$, given that one solution is $x = 3$.

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -4 & -24 \\ & & 3 & 12 & 24 \\ \hline & 1 & 4 & 8 & 0 \end{array}$$

$$(x-3)(x^2+4x+8) = 0$$

$x - 3 = 0$ OR $x^2 + 4x + 8 = 0$

$x = 3$ (given)

$$\frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 8}}{2}$$

$$\frac{-4 \pm \sqrt{16-32}}{2}$$

$$\frac{-4 \pm \sqrt{-16}}{2}$$

$$\frac{-4 \pm 4i}{2}$$

$$\boxed{\begin{array}{l} x = -2+2i \\ x = -2-2i \end{array}}$$

(can write as $-2 \pm 2i$)

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11. Let $R(x) = x^3 + x^2 - 24x$. Then $R(2) = -36$. For what other values of x is $R(x) = -36$?

$$x^3 + x^2 - 24x = -36$$

$$x^3 + x^2 - 24x + 36 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -24 & 36 \\ & & 2 & 6 & -36 \\ \hline & 1 & 3 & -18 & 0 \end{array}$$

$$(x-2)(x^2+3x-18) = 0$$

$x - 2 = 0$ OR $x^2 + 3x - 18 = 0$

$x = 2$ (given)

$$(x+6)(x-3) = 0$$

$x + 6 = 0$ OR $x - 3 = 0$

$x = -6$ or $x = 3$

(Could also use the quadratic formula instead of factoring)

12. What is a polynomial with single roots at $x = 3$ and $x = -2$, and a double root at $x = 1$? Write the polynomial in standard form (e.g. $a_n x^n + \ldots + a_1 x + a_0$) for full credit.

$$(x-3)(x+2)(x-1)^2$$

$$(x^2 - x - 6)(x^2 - 2x + 1)$$

	x^2	$-x$	-6
x^2	x^4	$-x^3$	$-6x^2$
$-2x$	$-2x^3$	$2x^2$	$12x$
1	x^2	$-x$	-6

$$\boxed{x^4 - 3x^3 - 3x^2 + 11x - 6}$$

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13. Find all real and complex solutions to $x^4 - 25x^2 + 60x - 36 = 0$; given that two solutions are $x = -6$ and $x = 1$. (Be sure to read the polynomial carefully, it is 4th degree with no cubic term).

-6	1	0	-25	60	-36
		-6	36	-66	36
1	1	-6	11	-6	0
		1	-5	6	
	1	-5	6	0	

$$(x+6)(x-1)(x^2-5x+6) = 0$$

$x+6=0$ OR $x-1=0$ OR $x^2-5x+6=0$

$(x-2)(x-3)=0$

$\boxed{x=-6}$ (given) $\boxed{x=1}$ (given) $\boxed{x=2 \text{ or } x=3}$

(Could also use the quadratic formula instead of factoring)

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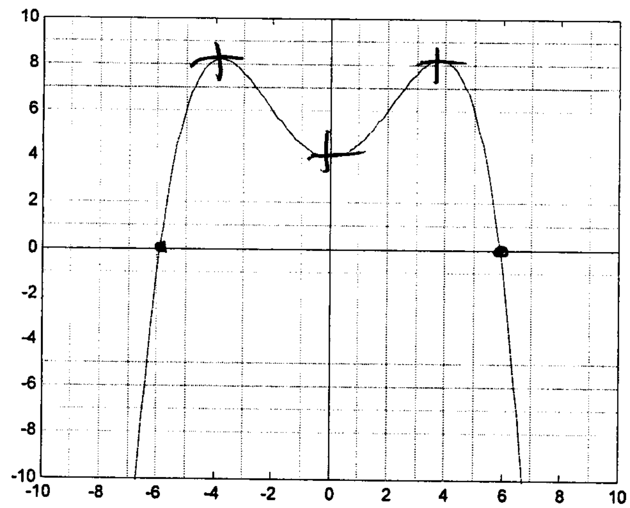
14. Answer the following questions about the polynomial graphed at the right. You may assume nothing interesting happens outside the window shown.



How many real roots does the polynomial have?

2

How many turning points?

3

Is the leading coefficient positive or negative?

Negative ↓ ↓ at $\pm\infty$

Is the constant term positive or negative?

Positive Crosses y-axis above x-axis

The least possible degree of this polynomial is?

4 (since 3 turning points, ~~[illegible]~~)

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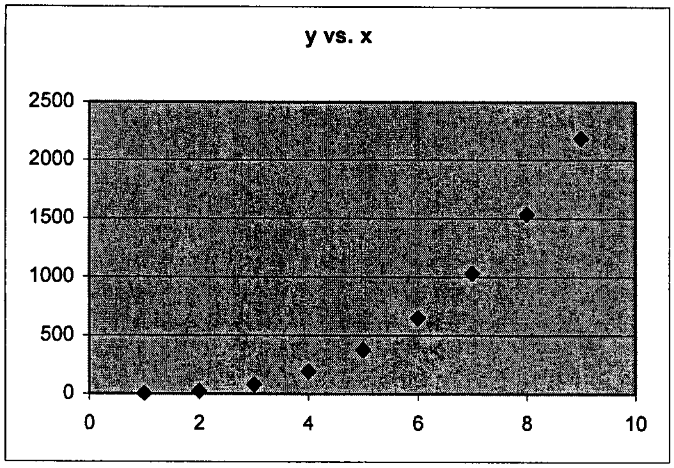
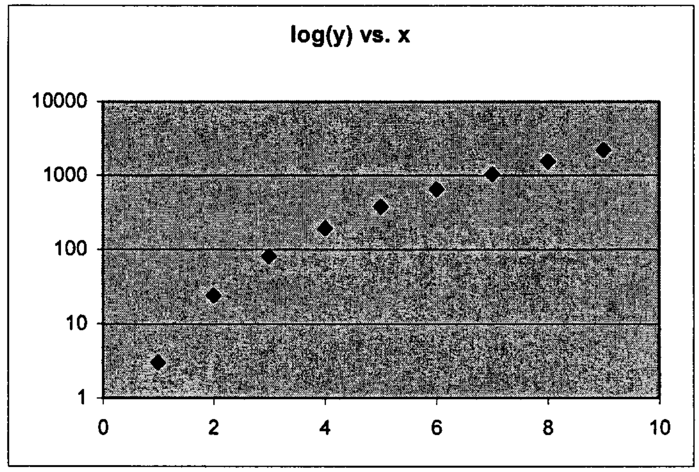
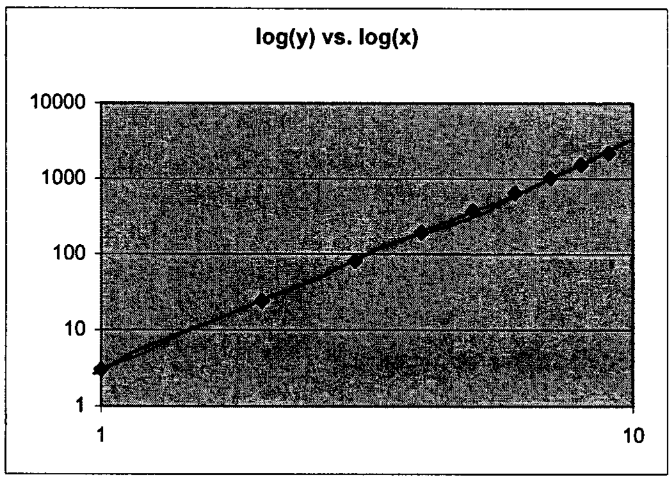
15. A data set is plotted below on standard, semi-log, and log scales. What type of function would be best for modeling this data? Explain your reasoning.







The short form of the answer is that since the data forms a line in the log(y) vs. log(x) view, a power law, $f(x)=ax^r$, is the appropriate model. (The short form does get full credit)

A longer answer can be found on the solutions to last fall's exam. Just saying power law without providing reasoning doesn't get full credit.