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Key

Math 100 Sections E/F
Exam 1
Spring 2007

1. Fill in the function table for $f(x) = \frac{x^2 + 3x + 2}{2}$.

x	-2	-1	0	1	2
f(x)	0	0	1	3	6

$$\frac{(-2)^2 + 3(-2) + 2}{2} = \frac{4 - 6 + 2}{2} = 0$$

$$\frac{(-1)^2 + 3(-1) + 2}{2} = \frac{1 - 3 + 2}{2} = 0$$

$$\frac{0^2 + 3(0) + 2}{2} = \frac{2}{2} = 1$$

$$\frac{1^2 + 3(1) + 2}{2} = \frac{1 + 3 + 2}{2} = 3$$

$$\frac{2^2 + 3(2) + 2}{2} = \frac{4 + 6 + 2}{2} = 6$$

2. Evaluate $R(z) = \frac{aY}{z-4a}$ at $z = 5a$.

$$R(5a) = \frac{aY}{5a-4a} = \frac{aY}{a} = Y$$

3. Solve for x: $13x - 2 = 5x + 14$

$$\begin{array}{r} -5x \quad -5x \\ 13x - 2 = 5x + 14 \\ \hline 8x - 2 = 14 \\ +2 \quad +2 \\ \hline 8x = 16 \end{array}$$

$$x = \frac{16}{8} = 2$$

4. Suppose $x = 4$ solves $ax - 6 = 3x + a$. What is a ?

$$\begin{array}{r} 4a - 6 = 3(4) + a \\ 4a - 6 = 12 + a \\ -a \quad -a \\ \hline 3a - 6 = 12 \\ +6 \quad +6 \\ \hline 3a = 18 \end{array}$$

$$a = \frac{18}{3} = 6$$

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5. Graph the line $5x + 2y = 10$. One Method: If $x=0$ $2y=10 \rightarrow y=5$
If $y=0$ $5x=10 \rightarrow x=2$

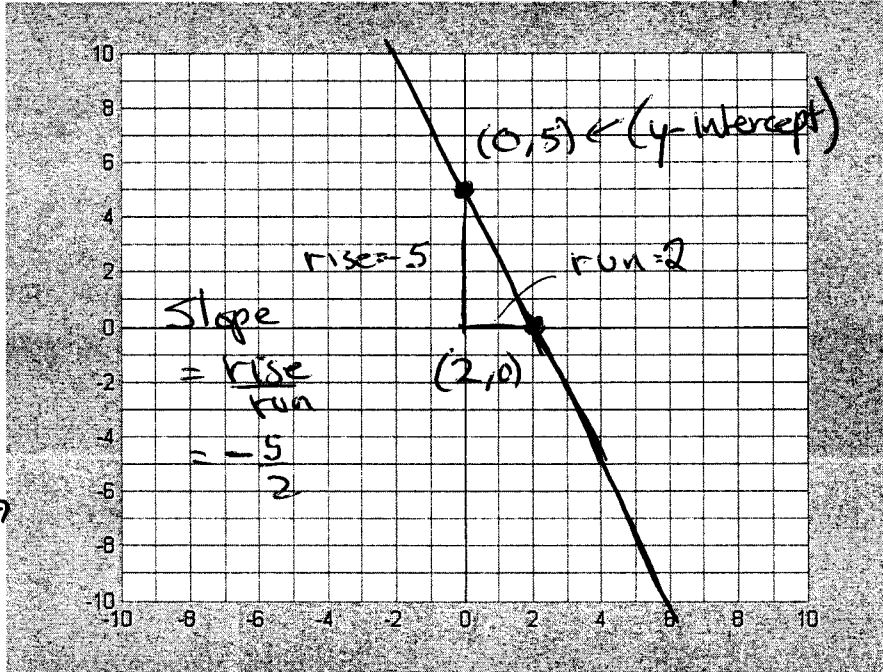
Alternative Method

$$5x + 2y = 10$$

$$2y = -5x + 10$$

$$y = -\frac{5}{2}x + 5$$

Draw line with
y-intercept = 5 \rightarrow
and slope $-\frac{5}{2}$

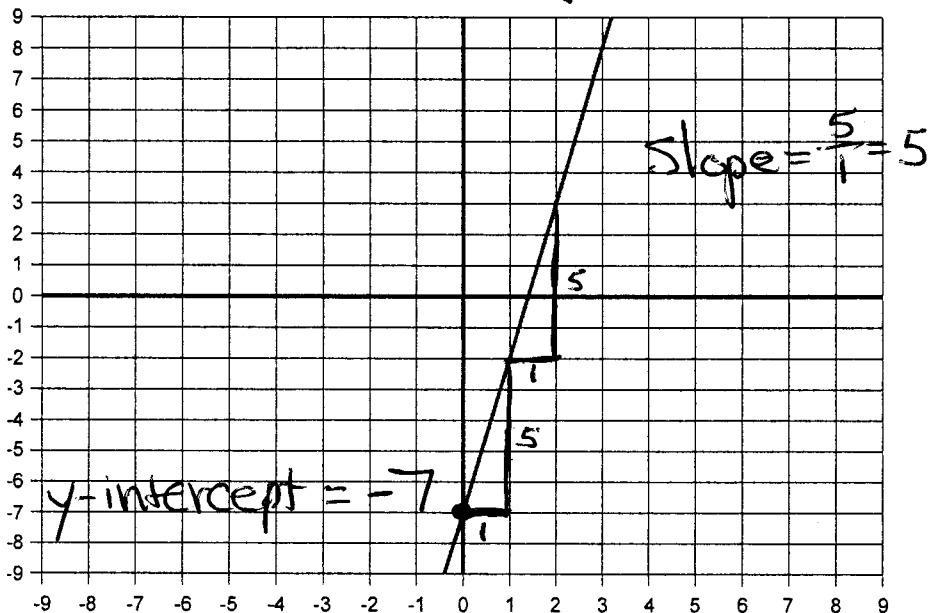


point (2, 0) point (0, 5)

Draw line through these two points

6. What is the equation of the line shown in the following graph?

$$y = 5x - 7$$



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7. Suppose a line passes through the points $(-4, 1)$ and $(6, 3)$. What is another point on the line?

Slope $\frac{3-1}{6-(-4)} = \frac{2}{10} = \frac{1}{5}$

Point-Slope Form $(y-3) = \frac{1}{5}(x-6)$

$$y-3 = \frac{1}{5}x - \frac{6}{5}$$

$$y = \frac{1}{5}x + \frac{9}{5}$$

$$\left(-\frac{6}{5} + 3 = -\frac{6}{5} + \frac{15}{5} \right. \\ \left. = \frac{9}{5} \right) \rightarrow$$

$$x=1$$

$$y = \frac{1}{5} \cdot 1 + \frac{9}{5}$$

$$= \frac{1}{5} + \frac{9}{5} = \frac{10}{5} = 2$$

Point $(1, 2)$

There are infinitely many correct answers and there are other correct techniques to find these answers

8. Simplify (solve) the inequality $2x+5 \leq 7-5x$

$$\begin{array}{r} +5x \quad +5x \\ 7x+5 \leq 7 \\ -5 \quad -5 \end{array}$$

$$7x \leq 2$$

$$\boxed{x \leq \frac{2}{7}}$$

9. Simplify (solve) the inequalities $x-2 < 3x+2 < 8-3x$

$$x-2 < 3x+2$$

$$\begin{array}{r} -x \quad -x \\ -2 < 2x+2 \\ -2 \quad -2 \end{array}$$

$$-4 < 2x$$

$$-2 < x$$

(Put them together again)

$$\boxed{-2 < x < 1}$$

$$3x+2 < 8-3x$$

$$\begin{array}{r} +3x \quad +3x \\ 6x+2 < 8 \end{array}$$

$$6x+2 < 8$$

$$\begin{array}{r} -2 \quad -2 \\ 6x < 6 \end{array}$$

$$6x < 6$$

$$x < 1$$

(Two different inequalities written in this one line)

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10. If possible, solve the system for x and y

$$\begin{array}{r} 2x + y = 4 \\ -4x - 2y = 6 \end{array}$$

add

$$\begin{array}{r} 2x + y = 4 \\ -4x - 2y = 6 \\ \hline 0 = 14 \end{array}$$

\times

No solutions

11. If possible, solve the system for x and y

$$\begin{array}{r} -3x + 7y = 1 \\ 6x - 10y = 2 \end{array}$$

add

$$\begin{array}{r} -6x + 14y = 2 \\ 6x - 10y = 2 \\ \hline 4y = 4 \\ y = 1 \end{array}$$
$$\begin{array}{r} -3x + 7 = 1 \\ -3x = -6 \\ x = 2 \end{array}$$

~~check~~ $x = 2, y = 1$

Check $6 \cdot 2 - 10 \cdot 1 = 12 - 10 = 2 \checkmark$

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12. Suppose bus ridership R and population P in a town are related by the equation $2P - 80R = 60,000$. For every 1,000 new people who move to town, how much will bus ridership increase?

$$2P - 80R = 60,000$$

$$2P = 60,000 + 80R$$

$$2P - 60,000 = 80R$$

$$\frac{1}{40}P - 750 = R$$

Slope = $\frac{1}{40}$ so for increase in P of 1000
there is an increase in R of $\frac{1}{40} \cdot 1000 = \boxed{25}$

13. A business depreciates its manufacturing equipment using a straight-line depreciation over 25 years. If the initial value of the equipment is \$200,000 and its value after 25 years will be 0, what is the depreciated value after 15 years?

Year 0 Value 200,000 (0, 200,000)

Year 25 Value 0 (25, 0)

Form the line through (0, 200,000) and (25, 0)

$$\text{Slope} = \frac{200,000 - 0}{0 - 25} = \frac{200,000}{-25} = -8,000$$

y-intercept is 200,000 (since we are given value is 200,000 when year is 0)

$$\text{So Value} = -8,000 \text{ year} + 200,000$$

$$\begin{aligned} \text{If year} = 15 \quad \text{Value} &= -8,000 \cdot 15 + 200,000 \\ &= -120,000 + 200,000 \\ &= \boxed{\$80,000} \end{aligned}$$

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14. If student Joe Schmoie has a course average score above 89, he will earn an A in the course (89 is a B, anything higher is an A). Suppose his scores are 79, 96, and 84 and his teacher said the final exam is worth twice the weight of the other exams. Write an inequality for the range of scores on the final exam which will result in Joe earning an A.

Let x = score on final.

since final twice the weight

$$\frac{79 + 96 + 84 + 2x}{5} > 89$$

$$2x > 186$$

$$\boxed{x > 93}$$

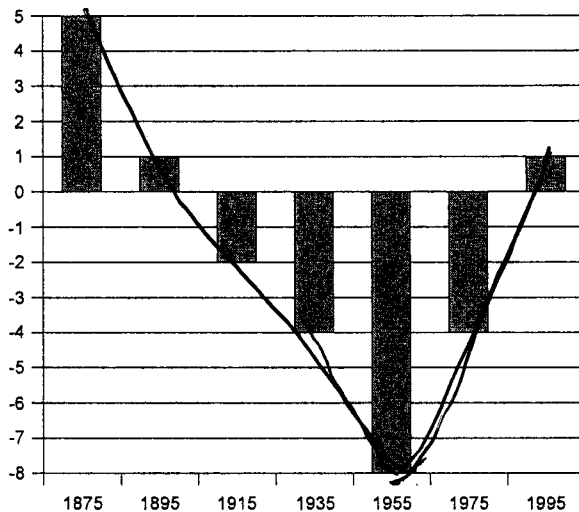
since 3 earlier exams + 2 for final

$$\frac{259 + 2x}{5} > 89$$

$$\begin{array}{r} 445 \\ - 259 \\ \hline 186 \end{array}$$

$$259 + 2x > 445$$

15. A linear model is developed to predict the number (in millions) of women in the workforce between 1875 and 1995. Comparing the model values to the observed data leaves the following residuals. Based on these residuals, does a linear model appear to be a good choice for this data? Write one or two sentences explaining why or why not.



The residuals form a pattern and not a random sequence of values. This suggests a linear model is a poor choice for this data.

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16. An investor owns two rental properties worth a total of \$300,000. Every year, she collects rent equal to 5% of the worth of the first property and rent equal to 7% of the worth of the second property. Her total annual rent from the two properties is \$17,200. How much is each property worth?

x = Value of first property

y = Value of second property

Total Value: $x + y = 300,000$

Total Rents: $.05x + .07y = 17,200$

$\times 20$ $\rightarrow x + y = 300,000$ (\leftarrow subtract this)

$\rightarrow x + 1.4y = 344,000$

$$0.4y = 44,000$$

$$y = \frac{44,000}{0.4} = 110,000$$

$$x + 110,000 = 300,000$$

$$x = 190,000$$

1 st property	\$190,000
2 nd property	\$110,000

check

$$.05(190,000) + .07(110,000)$$
$$= 9500 + 7700$$
$$= 17,200 \checkmark$$