Name: ____

Recitation: ____

Describe the Squirt Studio

This sheet includes both instruction sections (labeled with letters) and problem sections (labeled with numbers). Please work through the instructions and answer the questions in the problem sections. Turn in your answers to the *italicized prompts*, written in complete sentences to the appropriate studio box. You may turn in your work on this sheet.

We fit lines to data in the second studio. Unfortunately, as we saw with the population data, lines don't fit all data sets well. In this lab we will look at a situation where a parabola is the appropriate curve to describe the data, and go through how to find a quadratic function that models such a data set.

A. Open your web browser and go to the main course website. Click on the link Describe the Squirt, which will take you to the URL:

http://mathdl.maa.org/images/upload_library/4/vol2/liteapplets/JOMA_article/water_fountain.html

You will see a squirt, that is, a snap-shot of water flowing from a fountain. We are going to describe the shape of the water's flow. The shape resembles a parabola, and so we will try to model it with a quadratic function. We have learned that a quadratic function is determined by its vertex (h,k) and another distinct point on the graph. Together they determine the equation of the parabola, $y = a(x-h)^2 + k$. To model the shape of the water with this equation, we first need data describing its shape.

B. To collect the data we need to plot points. Click on a point on the picture of the squirt and the coordinates will be listed at the right. If you click on the blue "Mark point" button, the applet will draw a small black triangle (black diamonds on a Mac) at the point so you can see which points you marked. The black "Clear points" button will erase all these black triangles.

1. Click on the vertex of the parabola and record the coordinates in the form below. Then click on another point on the parabola (preferably well away from the vertex) and enter that point into the form as well. *Solve for the coefficients of the parabola in the form* $y = a(x-h)^2 + k$. Show your work in the space at the top of the next page.

	Coordinates
Vertex	
Another Point	

C. Enter the quadratic function you just found into the box labeled "The function f(x)" on the webpage. You must use * for multiplication on this page, and ^ for raising to a power. Also note that you may enter the formula in any correct form you wish, not only the standard form. Next, enter the lower and the upper bounds for *x*. To determine these, click on the left most part of the squirt and read off the *x* value and then click on the right most part of the squirt and read off the *x* value. Hit ENTER, or click on the button labeled "Try it!!"

2. *How close is the model to the actual shape?* In other words, does it appear to be a good fit or not. If it is not a good fit, try a different second point, further away from the vertex. Also, you may have rounded too far; that is, left off significant digits. Be sure to have at least 2 significant digits (non-zero digits after the decimal point, in this case) for the value of *a*. Note, if you modify the formula and try again, it will modify the image in a new window. However, it may not bring this new window to the foreground; which means you may have to bring the window to the front. *If you had to make adjustments (change a point, include more digits, etc.), then write what you did to get a good fit and the new equation.*

If we just pick out a vertex and one other point, we can draw a parabola. But our answer will be very sensitive to the points we choose. Clicking just a little bit off from the vertex may give us a poor fit. As in the earlier studio about fitting lines, we can do better (or at least safer) by picking a collection of points along the parabola and letting the spreadsheet find the best fit to the whole set of points at once.

D. As before, we will mark points on the squirt. But this time, we will do so from the far left all the way to the right, getting at least 12 points.
   a. Click on the black button to clear the points that may be already marked.
   b. Click on the image at a point on the far left.
   c. Click on the blue button to mark the point.
   d. Repeat steps b and c to mark twelve distinct points on the image of the water.
   e. Click on the red button to list the coordinates of the points you have marked.

E. A "Java Applet Window" will open when you click the red button to list the points. Scroll down and you will see an array of points. The numbers in the first column are the x-coordinates and the numbers in the second column are the y-coordinates. Highlight and copy these numbers. To copy the number type CTRL-C (Apple-C on a Mac) after they are highlighted.

F. Open a new Excel sheet. Put the label x in cell A1 and the label y in cell B1. Now paste the data set into the sheet so the *x* values start at cell A2 and the *y* values start at cell B2. Click and drag to select the data points, the click on the Chart Wizard to draw an XY graph of the data. Right-click on one of the data points and select "Add Trendline." On the Type tab, set the type of the trendline to Polynomial with Order 2. On the Options tab, select "Display equation on chart" and "Display R-squared value on chart." Then click OK.

G. With the trendline plotted in Excel, you can see how well it fits the specific points you picked. But what we are really interested in is how well it fits the picture. Enter the formula for the quadratic model into the website and see how well it fits. Remember to make sure you have enough decimal places for the coefficients *a*, *b*, and *c*. Also remember to enter in the largest and smallest values of *x* into the website, and to use the * key for multiplication.

3. *What is the function the spreadsheet produced for the curve? How well does this function fit the actual picture of the squirt?*

4. *What is the vertex of the function produced by the spreadsheet? How does this compare to where you placed the vertex by "eye-balling" it?*

5. Write both the function you found in problem 2 and the function you found in problem 3 in the same format, either both in vertex form, $y = a(x-h)^2 + k$, or both in standard form, $y = ax^2 + bx + c$. *How do the two functions compare?*

BONUS

We've seen that if we know the vertex and one additional point, we can find a parabola. But what if we don't know the vertex? If we know (for certain) any *three* points on the curve, we can find the formula for the parabola. We do this by plugging the three points into the formula for the parabola, and then realizing we have 3 linear equations in 3 unknowns. We can solve this using the same sorts of techniques as we used in solving 2 linear equations in 2 unknowns.

Suppose we are given three points on an unknown quadratic curve (x_1, y_1), (x_2, y_2), and (x_3, y_3). We get three linear equations in the three unknowns a, b, and c by substituting the known x and y values from the three points into the quadratic equation $y = ax^2 + bx + c$ to get

$$\begin{aligned} y_1 &= ax_1^2 + bx_1 + c \\ y_2 &= ax_2^2 + bx_2 + c \\ y_3 &= ax_3^2 + bx_3 + c \end{aligned}$$

For example, if we are given the points (-1,6), (0,1), and (2,3), then we have the equations

$$\begin{aligned} 6 &= a - b + c \\ 1 &= \qquad\quad c \\ 3 &= 4a + 2b + c \end{aligned}$$

6. *Solve this system of equations to find the parabola passing through the three points,* (-1,6), (0,1), and (2,3).

7. Suppose instead of using standard form, we used vertex form, $y = a(x-h)^2 + k$. Just as above, we could plug in three points, (-1,6), (0,1), and (2,3), to get three equations in the unknowns *a*, *h*, and *k*. *Write the three equations for a, h, and k you get by substituting these three points into the vertex form. Why would these equations be more difficult to solve that the equations for problem 6?*

Since vertex form is easier to use for drawing graphs (and also for solving equations) than standard form, you may have wondered why we bothered with standard form at all. This should give you an idea of why standard form can be useful. Of course, the situation is more complicated still if you are looking for the best fit through a dozen or more points, but the fundamental point will remain, in fact the greater complexity will make it even more true. Using standard form leads to much simpler equations when you are trying to find the parameters of a parabola and you *don't* have the vertex specified.

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