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Roots of Quadratic Equations Studio

We've discussed finding the vertex of a parabola. If we have a quadratic in the form $y = a(x - h)^2 + k$, then the vertex is at the point (h,k), indeed the reason for writing the function in the form is exactly that it lets us spot where the vertex is easily. Now the vertex is interesting because it is the lowest (or highest) point on the parabola. But sometimes we are interested in when a quadratic function takes some particular value. In this studio, we will look at how to find when a quadratic function takes the value 0. We discussed the quadratic formula in lecture and you can read a justification of this formula on pp. 172-173 of the text. In this lab we will look at patterns for how zeros are related to the vertex of a parabola. The quadratic formula is related to these patterns, though we won't go into details in this lab.

Go to the class web site and download the file m100qestudio.xls. This file uses "macros" and these might be disabled, depending upon the security settings you have for Excel. If you are unable to use the sliders, open Excel and choose Tools-Macro-Security and set the security level to medium. Close and then re-open the spreadsheet and you should be able to use the sliders. When you open the spreadsheet, be sure to choose "Enable Macros" if you are asked.

This file shows the graph of the parabola $y = a(x - h)^2 + k$ on a fixed window, [-10, 10] x [-10, 10]. The coefficients a, h, and k, are adjusted by moving three sliders at the top of the page. The specific values of a, h, and k are listed to the left of the sliders. The zeros of the function, that is the x values where the function takes the value 0, or, in geometric terms, the places where the graph crosses the x-axis, are given below the graph. Move the sliders around. Observe how changing h and k changes the vertex while changing a changes how spread out the parabola is.

1. Move the sliders to set the value of $a = 1$ and $h = 0$. Just move the slider for k around. *Fill in the missing values in the table below.* **Note that the slider for k won't move far enough to find the answers for the last row.** So how are you supposed to answer that? Look for a pattern from the answers you find from the first three rows and use that pattern to deduce what values must go in the last two row.

K	**Zeros**
	-1 and 1
	-2 and 2
	-3 and 3
	-4 and 4
-25	

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2. Now move the sliders for both *h* and *k* (while *a* is still 1). *Fill in the missing values in the table below.* You will find it helpful to observe that changing *k* changes how far apart the two zeros are without changing the midpoint between the two zeros, while changing *h* slides the zeros on the axis, without changing how far apart the zeros are. Once again, you won't be able to push the slider far enough to get the last row, but you should be able to answer the question based on the patterns you find from the first four rows.

H	***k***	**Zeros**	**Midpoint of the Zeros**	**Distance Between Zeros**
		3 and 5		
		-3 and 1		
		4 and 4 (this is called a *double root*)		
		1 and 7		
		-7 and 3		

3. *Describe the pattern for the relationship between the zeros and the values of h and k.* In particular, how are *h* and *k* related to the midpoint of the two zeros and how far apart the zeros are?

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4. *Solve* $(x - h)^2 + k = 0$ *for x in terms of h and k. Describe how this formula matches the pattern you found for h in the previous problem.* In particular, note where the midpoint of the zeros is according to the formula and how far apart the zeros are according to the formula.

5. You may have noticed when you moved the sliders around that sometimes the zeros were listed as #NUM! This means that there were no zeros, because the parabola never crossed the *x*-axis. *For what values of h and k are there no zeros? For what values of h and k is there a double root (two equal zeros)?Does the answer change if you slide a to be -1 instead of 1?*

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Bonus:

Now that we've worked out the easier monic case (with $a = 1$), we're ready to tackle the full situation where we can have a take other values.

6. Now you should move all three sliders. *Fill in the missing values in the table below.*

a	***h***	***k***	**Zeros**
0.5			-1 and 1
0.5			-2 and 2
3			-1 and 1
3			-2 and 2
-2			-1 and 1
-2			-2 and 2
0.5			3 and 5
0.5			-3 and 1
3			0 and 2
3			1 and 5
-2			-3 and -1
-2			2 and 6

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7. *Solve* $a(x - h)^2 + k = 0$ *for x in terms of a, h and k.*

8. *Describe how this formula matches the patterns you found doing problem 6.*