

Name: _____

Studio on Logistic Models -- Extra Credit

Logistic functions are important in many applications. As noted in the text (section 3.7), many things, such as spread of a rumor or a cold, or adoption of a new product, are best modeled by an “S-shaped” curve, where you start off slowly, then growth accelerates as enough people know the rumor or use the product for lots of new people to be attracted, and then things finally level off as the situation reaches “saturation” (e.g. everyone has already heard the rumor). Note that the text points out logistic functions follow such a curve, but doesn’t give you any reasoning to see why the funny formula $y = \frac{c}{1 + ae^{-bx}}$ produces such a curve or what role the parameters a , b , and c play. The next few exercises below will help you gain a better, if still not complete, understanding of why logistic functions behave the way they do. For more details, take calculus.

1. Consider the logistic function $f(t) = \frac{100}{1 + 2e^{-0.5t}}$.

a. What value does $e^{-0.5t}$ approach as t goes to $+\infty$ (i.e. what is the asymptote of $e^{-0.5t}$ as t goes to $+\infty$)?

b. If you substitute the value you found in part (a) in place of $e^{-0.5t}$ in the formula for $f(t)$, what value does $f(t)$ take?

c. From your answer to (b), what is the asymptote of $f(t)$ as t goes to $+\infty$?

Name: _____

2. Consider the logistic function $f(t) = \frac{250}{1 + 4e^{-2t}}$

- a. What happens to the value of e^{-2t} as t tends to $-\infty$?

- b. From your answer to (a), what can you say about the value of $1 + 4e^{-2t}$ as t tends to $-\infty$?

- c. From your answer to (b), can you explain why $f(t)$ has an asymptote of $y = 0$ as t tends to $-\infty$?

3. Consider the logistic function $f(t) = \frac{300}{1 + 5e^{-t}}$

- a. What is $f(0)$?

- b. Using the same reasoning as in problem 1, what will the asymptote of $f(t)$ be as t tends to $+\infty$?

- c. Since we know the logistic function has an asymptote at $y = 0$ as t tends to $-\infty$ and you worked out the value at $t = 0$ and the asymptote as t tends to $+\infty$, sketch the curve. In trying to fit in both asymptotes, you should see why logistic functions give “S-shaped” curves.

Name: _____

Fitting a Logistic Model to Data

In addition to the situations described earlier, logistic models can also be applied to a variety of situations in agriculture. In particular, one can use a logistic model to describe the growth of certain plants or fungi. The example we study here is the growth of a sunflower, which Reed and Holland studied in 1919 (source: <http://www.seattlecentral.edu/qelp/sets/009/009.html>).

Download the data file `logisticstudiodatas08.xls` from the course website. The data lists the height of a sunflower (starting from a seed) in intervals of seven days.

Exercises

A. Create a scatter plot of the data in columns A and B as you normally would using the Chart Wizard icon.

4. *What kind of pattern do the data points follow?*

Hopefully you noticed the data points roughly fall on an s-shaped curve. A logistic model would be appropriate for this situation, but in Excel, we don't have the option of adding a logistic trendline. We can, however, use a feature called Solver to help us find values of a , b , and c that would make a good logistic model.

How does Solver work? Solver tries to optimize (this means find a maximum or minimum) a target value by adjusting values in what are referred to as 'changing cells,' specified by the user. In this example, the 'changing cells' will be the three parameters in the logistic model: a , b , and c . The target value will be the sum of the squared values of the error between the given data and the model we are trying to fit. We want to minimize the target value (this is the basic idea behind the Least Squares Method, which you will run into again if you take Stat 325).

We need to choose initial values for the three parameters – you will make realistic guesses for these values. Excel will adjust these values until it finds a solution that minimizes the target value. In a nutshell, Solver helps us find values of a , b , and c that are part of the logistic model formula.

Name: _____

B. The next step is to enter values into the ‘changing cells.’ In cell E1, enter ‘a’, in cell E2, enter ‘b’, and in cell E3, enter ‘c.’ Then, in cell F1, F2, and F3, enter reasonable guesses as initial values for a, b, and c. For instance, remember that ‘c’ represents a limiting value (as you noticed in the above exercises). In context of this situation, ‘c’ represents the value at which the sunflower’s height levels off. Based on the data, you might choose $c=275$, so enter 275 in cell F3. Similarly, you could enter .25 in cell F2, and 100 in cell F1. Your spreadsheet should now look like this:

	A	B	C	D	E	F
1	Days	Height (cm)			a	100
2	0	0			b	.25
3	7	17.93			c	275
4	14	36.36				
5	21	67.76				
6	28	98.1				
7	35	131				
8	42	169.5				
9	49	205.5				
10	56	228.3				
11	63	247.1				
12	70	250.5				
13	77	253.8				
14	84	254.5				
15						

C. In cell C2, enter the following formula: $=($F$3)/(1+($F$1)*EXP(-$F$2*A2))$. Then click and drag from cell C2 down to cell C14, copying the formula. You have now entered the formula of the logistic model with the parameters we have initially set. In this formula, A2 refers to the number of days, which is the ‘x’ value in the logistic model formula.

D. Next, in cell D2, enter the following formula: $=(C2-B2)^2$. Then, click and drag from cell D2 down to D14, copying the formula. In this formula, you are calculating the square of the error between the observed data and the model we are trying to fit.

E. Now, in cell D15, enter the following formula: $=SUM(D2:D14)$. This is simply the sum of all the values in D2 through D14. Cell D15 is the target cell, which we are trying to minimize (the least squares method discussed earlier.)

F. We are now ready to use Solver. Go to the Tools menu at the top, and click on Solver. (If you don’t see Solver, click on Add-Ins, and then you will have the option to install Solver. This will just take a few minutes. After it is installed, go back to the Tools menu and click on Solver, which you should now see.)

Name: _____

A box called 'Solver Parameters' should come up. Inside the target cell box, enter \$D\$15. Then, choose the 'min' option. Then in box labeled 'By Changing cells,' enter \$F\$1:\$F\$3.

(By doing this, you are telling Solver to find a solution that minimizes the value in cell D15, and it will adjust the parameters in cells F1 through F3 until a solution is found.)

Now, click on 'Solve,' and you should get a message that Solver has found a solution. It will ask you if you would like to keep the Solver solution, and you should click on OK. The new solution is located in cells F1-F3. These are the values of a, b, and c that you plug into the

logistic model $y = \frac{c}{1 + ae^{-bx}}$.

5a) What values of a, b, and c did Solver find? What is the formula for the logistic model?

5b) Based on the model, at what height does the sunflower's growth level off?

5c) Using your model in part (5a), how long will it take for a sunflower to grow to a height of 50 centimeters? (Hint: plug in 50 for y, and solve for x. You will eventually have to use logarithms in your calculations.)