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Doubling Time Studio

To develop a better feel for how exponential functions grow, we will use a spreadsheet to play with compound interest. Open Excel and follow the steps below to analyze what happens if you put $1000 in the bank and let it sit and collect interest.

A. First we will label our columns. Enter "Period" in cell A1, "Interest" in cell B1, "Balance" in cell C1, and "Rate" in cell F1. Next, set up the initial values. Enter 0 in cell A2 under Period, 1000 in cell C2 under Balance, and .08 in cell F2 under Rate. Leave cell B2 under interest blank. The spreadsheet will be easier to read if we format the cells properly. Right click on the "B" at the top of column B to highlight the whole column. This will bring up a menu of options. Select "Format Cells…" and then select "Currency" (under the "Number" tab, which is usually the tab initially selected). You can choose several different formats for currency depending on your personal preferences. Use the same procedure to format column C for currency as well. Finally, right click on cell F2, and format that cell as a Percentage.

B. Now set up the formulas. Enter 1 in cell A3. Enter "=F2*C2" in cell B3. You can either type this formula in or build it using your mouse. Remember that if you click on a cell (say F2), you can press the "F4" key at the top of the keyboard to convert F2 to F2. Enter the formula "=C2+B3" into cell C3. The spreadsheet should now look like

Period	Interest	Balance			Rate
0		$1,000.00			8.00%
1	$80.00	$1,080.00			

Note that the computations here show how your money will grow if you receive 8% interest. At the start (period 0), you just have your initial deposit of $1000. But after 1 period (which might be a year or whatever time period specified in the agreement with the bank), you will receive 8% of your account as interest. In this case, you get 8% of $1000 which is $80. This is added to the balance in your account, so you now have $1080.

C. Now click and drag with your mouse to select cells A3 through C3. A box will appear around the cells, with a small square in the lower right corner. Click on the square and pull it down to extend the formulas down 50 rows or so. The first several rows of the spreadsheet should now look like the table at the top of the next page. You will probably want to double-click on the line between the gray C and D boxes at the top to resize column C to fit the large numbers you get at the end.

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Period	Interest	Balance			Rate
0		$1,000.00			8.00%
1	$80.00	$1,080.00			
2	$86.40	$1,166.40			
3	$93.31	$1,259.71			
4	$100.78	$1,360.49			
5	$108.84	$1,469.33			

1. *Fill in the table below to show when the balance in the account reaches the following amounts*. Note that you won't hit the amount on the nose, so list when you come closest (e.g. you won't hit $2000 exactly, but you will get very close after 9 periods with $1,999.00).

Balance	Period
$1000	0
$2000	
$4000	
$8000	
$16,000	
$32,000	

What pattern do you notice in how long it takes the value in the account to double?

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D. Since the rate of change (the amount of interest you receive) is proportional to the amount of money you have invested, this function is an example of exponential growth. You may find it useful to graph the values in column C (the balances) to see how this exponential growth curve looks. That it always takes the same length of time for the value of the investment to double is characteristic of exponential growth. The length of time it takes for the value to double is called the "doubling time." Now change the rate in cell F2 to see how the doubling time is affected by the interest rate.

2. Fill in the chart below

Interest Rate	**Doubling Time**
2%	
4%	
6%	
8%	
10%	
12%	

Can you find a pattern for the doubling time for the different rates?

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3. Suppose you invest $10,000. *How much will you have after 36 years if the doubling time is 9 years? How much will you have after 36 years if the doubling time is 6 years.* **Note that you don't need to use the spreadsheet to answer these questions.** From the previous question, you should see that it doesn't take a big change in the interest rate to change the doubling time from 9 to 6 years, but that such a small change in the interest rate can lead to large differences in how much money you make.

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4. To get a better sense of how exponential growth compares to other functions we've seen, let's look at a quadratic function and see how long it takes to double in value. Let $f(x) = x^2$. *Fill in the following table similar to that you did in problem 2.* **You don't need to use a spreadsheet to fill in this table, you should just solve for the values using algebra. Do note that you are solving for *x* given the value of x^2, not the other way around.**

f(x) = x²	***x***
1	1
2	
4	
8	
16	
32	
64	

Does it always take the same length of time for a quadratic function to double in value, like an exponential?

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E. Of course, not everything goes up. As discussed in the text, you can have exponential decay where the amounts are decreasing each time by an amount proportional to how much is left. You get exponential decay by taking a negative rate. Enter –4% in cell F2. Now the values are going down, so you aren't going to get a doubling time. But you will get a "half-life."

5. *Fill in the following table for the balance when the interest rate is –4%.* Note that you may need to pull your formulas down farther than before.

Balance	Period
$1000	0
$500 (1/2 of $1000)	
$250 (1/4 of $1000)	
$125 (1/8 of $1000)	
$62.50 (1/16 of $1000)	

What is the half-life of an exponential decay of 4%?

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<u>Bonus</u>

We have simplified things a little bit above. In practice, interest rates are listed as annual rates, but are paid at a different schedule. To illustrate, let's assume you invest $1000 at 100% annual interest (if you are ever offered a secure investments that pays100% annually, please let me know). If the bank pays you interest annually, then after 1 year you would have $2000, your original $1000 plus $1000 in interest. But the bank might compound your interest semi-annually. In that case, instead of giving you 100% interest once a year, it would pay you 50% interest twice a year, which would give you a total of $2,250. Or it might compound quarterly, in which case it gives you 25% interest four times a year.

6. *Fill in the chart below for how much you will have if you invest $1000 at 100% annual interest, compounded as listed.* You will probably need to pull your formulas down even further to complete this table.

Compounding	Balance after 1 year
Annual	$2,000
Semi-Annual	$2,250
Quarterly	
Monthly	
Weekly	
Daily	
Hourly	

By the way, in real life a bank that offers monthly compounding usually won't actually offer 1/12 of the interest every month but will offer 28/365 of the interest in February and 31/365 of the interest in March and so on. But for this assignment, just assume monthly compounding means 12 periods at 100%/12. The difference this makes is small.

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7. You can see that the more frequently your interest is compounded, the more money you will make. Surprisingly though, there is a limit to how much you can make at 100% interest, no matter how fast the account compounds. *What do you think is the limiting value for the balance for investing $1000 for 1 year at 100% interest, as the speed of compounding tends to infinity?* Look at the last value in the chart you filled out and you may be able to guess. (A hint that only makes sense if you went to lecture: Who was the seventh president of the United States?)