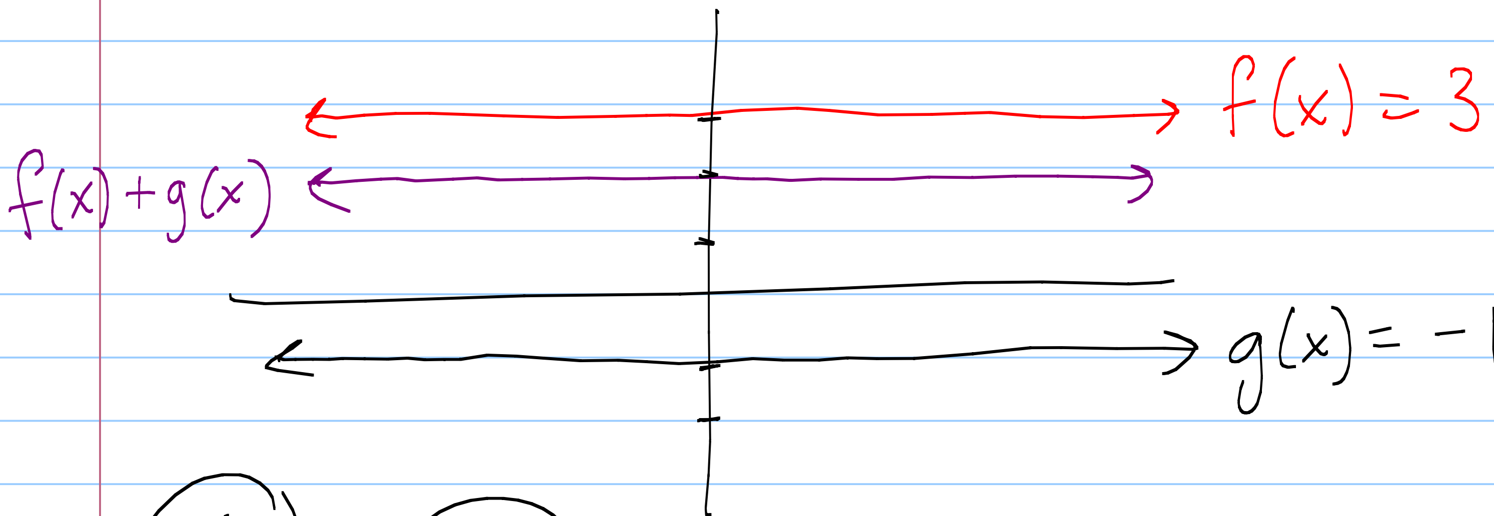
Section 2.6 - Combining Functions

Note Title 2/13/2008



$f(x) + g(x) \longrightarrow$ How would this look?

output or y-value

Combining Functions: Ex from online HW

$f(x) = 2x^2 + 6x + 5$

$g(x) = 3x^2 - 2x + 5$

$(f+g)(x)$

(A) Adding functions: $f(x) + g(x)$

$(2x^2 + 6x + 5) + (3x^2 - 2x + 5)$

$= 2x^2 + 6x + 5 + 3x^2 - 2x + 5$

$= \boxed{5x^2 + 4x + 10}$ $f(x) + g(x)$

(B) Subtracting Functions

<u>Why?</u> Profit = Revenue − Cost

$$P(x) = R(x) - C(x)$$

$$f(x) - g(x) = 2x^2+6x+5-(3x^2-2x+5)$$

$$= 2x^2+6x+5-3x^2+2x-5$$

$$= \boxed{-x^2+8x}$$

(c) Multiplying Functions

$f(x) \times g(x) = (2x^2+6x+5)(3x^2-2x+5)$

$f(x) \cdot g(x)$

$(fg)(x)$

	$2x^2$	$6x$	5
$3x^2$	$6x^4$	$18x^3$	$15x^2$
$-2x$	$-4x^3$	$-12x^2$	$-10x$
5	$10x^2$	$30x$	25

$= 6x^4 + 14x^3 + 13x^2 + 20x + 25$

Be careful

Diagonals don't match →

	$2x^2$	$5x$	1
x^2	$2x^4$	$5x^3$	x^2
1	$2x^2$	$5x$	1

"COMPOSITION"

Screen Saver → Expanding Circle.

2.5 cm/sec (radius ~~is~~ growing at this rate)

$r(t) = 2.5t$ (Radius function)

Area of Circle: $A = \pi r^2$

(Area Function) $A(r) = \pi r^2$

What is the area of the circle as a function of time?

$$\underbrace{A(r(t))}_{\text{Composition}} = \pi (r(t))^2 = \pi (2.5t)^2$$

$$= 6.25\pi t^2$$

$(A \circ r)(t)$

$3\square^2 - 2\square + 5$

<u>From Online HW:</u> $g(x) = 3\underline{x}^2 - 2\underline{x} + 5$

$h(x) = 2x + 3$

Find $g(\underline{h(x)}) = g\left(\underline{(2x+3)}\right)$

$g(\quad)$ $= 3(2x+3)^2 - 2(2x+3) + 5$

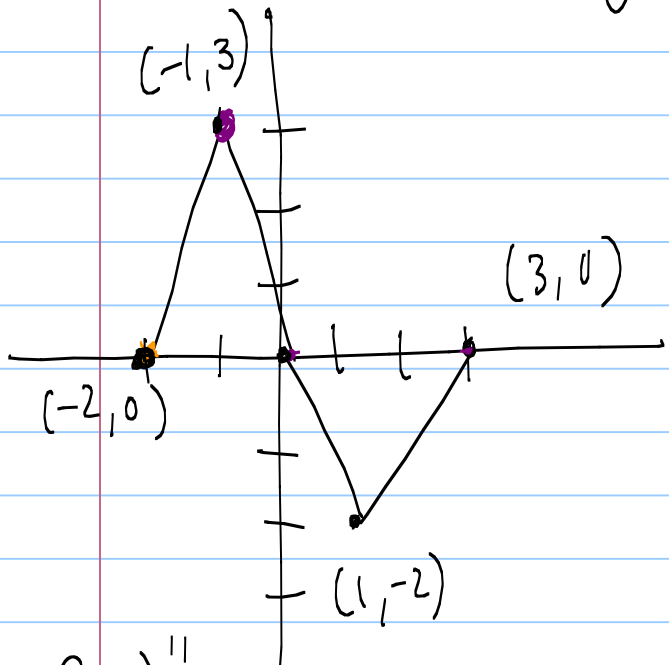
$= 3(2x+3)(2x+3) - 4x - 6 + 5$

$= 3(4x^2 + 12x + 9) - 4x - 1$

$= 12x^2 + 36x + 27 - 4x - 1$

$g(h(x)) = \boxed{12x^2 + 32x + 26}$

2.7/ Finding Inverse Functions

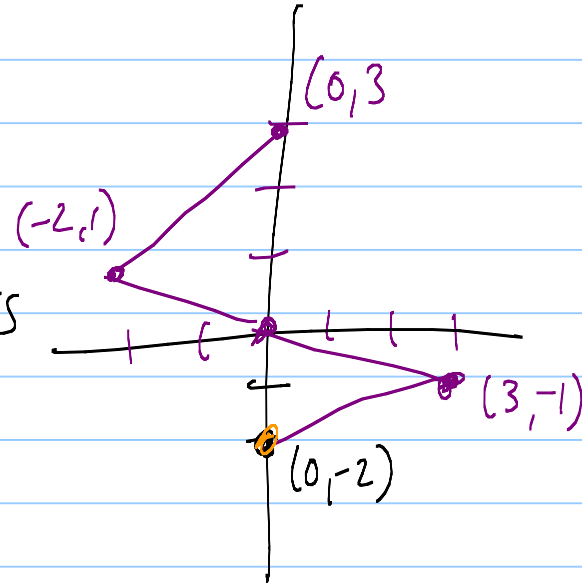


"f(x)"

Switch

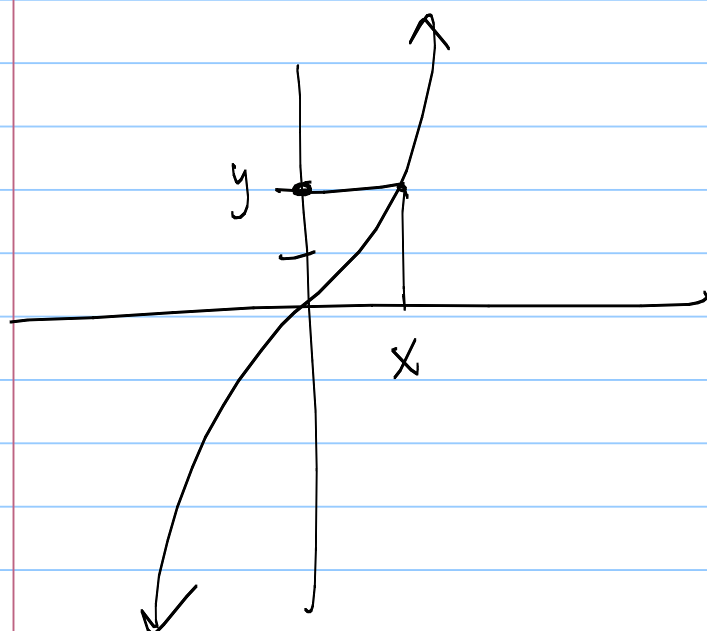
$\overset{\curvearrowright}{(x, y)}$

coordinates

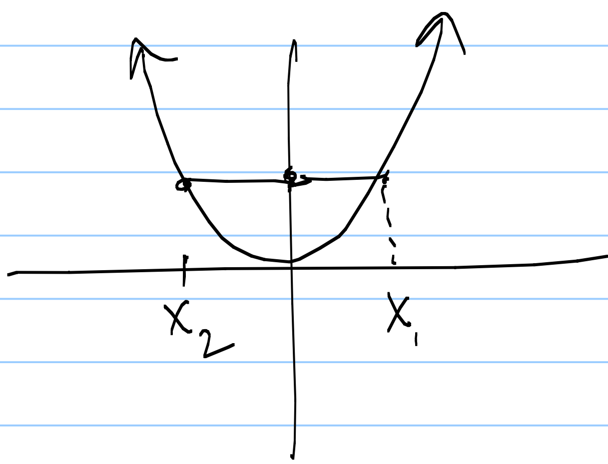


inverse "relation"

* For a function $f(x)$ to have an inverse, $f(x)$ must be "one-to-one"



← This is 1-1

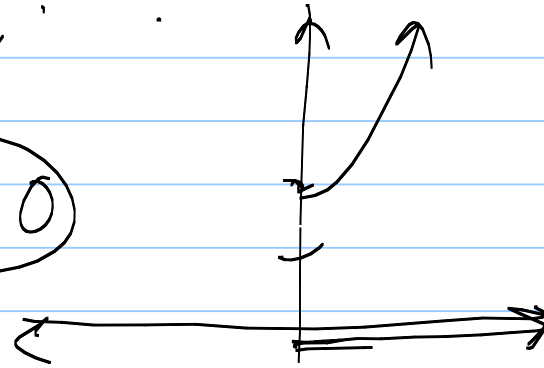


← Not 1-1

Horizontal Line Test: A horiz. line can only intersect a graph 1 time for it to be 1-1.
(If it's 1-1, then inverse exists)

Ex) Finding an inverse:

$f(x) = x^2 + 2$, $x \geq 0$



4-Steps

1. Change $f(x)$ to "y"
2. Switch x and y
3. Solve for y
4. Replace y with $f^{-1}(x)$

(1) $y = x^2 + 2$

(2) $x = y^2 + 2$

$-2 \qquad -2$

(3) $x - 2 = y^2$

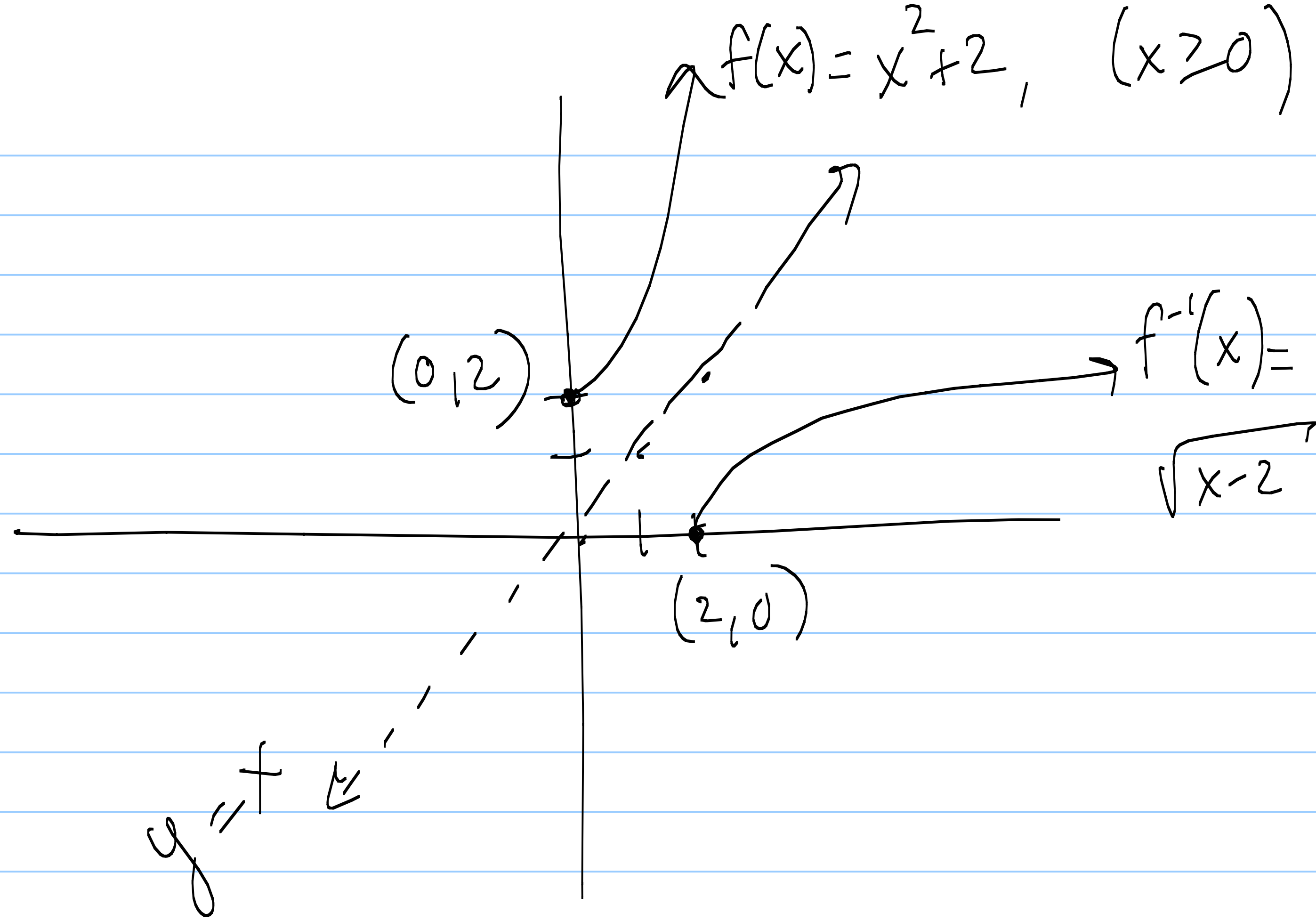
$\sqrt{x-2} = \sqrt{y^2}$

Keep (+) part

$\sqrt{x-2} = y$

(4) $f^{-1}(x) = \sqrt{x-2}$

This is the inverse of $f(x)$



~~(b)~~ $f^{-1}(x)$ is a reflection of $f(x)$ over the line $y = x$

Ex) Find the inverse of

$$f(x) = \sqrt{8x - 9} \qquad \left(x \geq \frac{9}{8}\right)$$

① $y = \sqrt{8x - 9}$

② $(x)^2 = \left(\sqrt{8y - 9}\right)^2$

③ $x^2 = 8y - 9$

$$\frac{x^2 + 9}{8} = \frac{8y}{8}$$

$$\frac{x^2 + 9}{8} = y$$

④ $f^{-1}(x) = \dfrac{(x^2 + 9)}{8}$

$$\frac{1}{8}x^2 + \frac{9}{8}$$