Name: KEY

Recitation Instructor and Time: ____________

Studio College Algebra – Exam 3
April 8, 2008

Please show all your work for full credit. Every problem is worth 5 points.

1. Rewrite the formula $y = 46x$ by taking the logarithm of both sides, and simplify your answer.

$\log y = \log(46x)$ or $\ln y = \ln(46x)$

$\log y = \log 46 + \log x$ $\ln y = \ln 46 + \ln x$

$\boxed{\log y = 1.6628 + \log x}$ or $\boxed{\ln y = 3.8286 + \ln x}$

2. Rewrite the formula $y = \dfrac{x^2}{20}$ by taking the logarithm of both sides, and simplify your answer.

$\log y = \log\left(\frac{x^2}{20}\right)$ or $\ln y = \ln\left(\frac{x^2}{20}\right)$

$\log y = \log x^2 - \log 20$ $\ln y = \ln x^2 - \ln 20$

$\boxed{\log y = 2\log x - 1.301}$ $\boxed{\ln y = 2\ln x - 2.9957}$

Name: ______________________________

3. If $\log(a) = 5.4$ and $\log(b) = 6.8$, what is $\log\left(\frac{a}{\sqrt{b}}\right)$?

$$\log\left(\frac{a}{\sqrt{b}}\right) = \log a - \log\sqrt{b}$$
$$= \log a - \log b^{1/2}$$
$$= \log a - \frac{1}{2}\log b$$
$$= 5.4 - \frac{1}{2}(6.8) = 5.4 - 3.4 = \boxed{2}$$

4. Solve $3 + 4e^x = 15$

$$\begin{array}{rcl} 3 + 4e^x &=& 15 \\ -3 & & -3 \\ \hline \end{array}$$

$$\frac{4e^x}{4} = \frac{12}{4}$$

$$e^x = 3$$

$$\ln e^x = \ln 3$$

$$x \underbrace{\cancel{\ln e}}_{(1)} = \ln 3$$

$$\boxed{x = \ln 3}$$

or $x \approx 1.0986$

5. Solve $6\ln(x-3) = 12$.

$$\frac{6\ln(x-3)}{6} = \frac{12}{6}$$

$$\ln(x-3) = 2$$

$$e^{\ln(x-3)} = e^2$$

$$x - 3 = e^2$$

$$\boxed{x = e^2 + 3}$$

or $x = 10.389$

Name: ____________________

6. What is the future value in 5 years of an initial investment of $600 at an annual interest rate of 4%, compounded quarterly?

# of periods: 4 times a year for 5 years

$4 \cdot 5 = 20$ periods

interest rate at each compounding: $\frac{.04}{4} = .01$

$$F.V. = P.V. (1+i)^n$$

$$F.V. = P.V. (1+.01)^{20}$$

$$F.V. = 600(1.01)^{20} = \boxed{\$732.11}$$

7. The population of a small town can be modeled by $P(t) = 3500e^{.023t}$, where $t=0$ represents the population in 1990. During what year did the population pass 4200?

$$\frac{4200}{3500} = \frac{3500e^{.023t}}{3500}$$

$$1.2 = e^{.023t}$$

$$\ln 1.2 = \ln e^{.023t}$$

$$\ln 1.2 = .023t \ln e$$

$$\frac{\ln 1.2}{.023} = \frac{.023t}{.023}$$

$$t \approx 7.9$$

$1990 + 7.9$

In 1997, the population passed 4200.

Name: ______________________________

8. The number of gizmos (in thousands) demanded each year is given by the formula $D(x) = 4 + 3\log(x+2)$, where x represents the number of years after 1980, and $x > 0$. How many gizmos were demanded in the year 1992?

1992 corresponds to $x = 12$.

$$D(12) = 4 + 3\log(12+2)$$
$$= 4 + 3\log(14)$$
$$\approx 7.438$$

$\boxed{\text{7,438 gizmos were demanded}}$

9. Find 2 possible 4^{th} degree polynomials with single roots at $x = 3$ and $x = -2$, and a double root at $x = 0$. Write the polynomials in standard form $a_n x^n + \ldots + a_1 x + a_0$ (in other words, multiply everything out). (Show your work, but put answers in the lines given below).

First answer: $x^4 - x^3 - 6x^2$

Second answer: $k \cdot x^4 - k \cdot x^3 - k \cdot 6x^2$ (where k is a nonzero constant). So any multiple of the 1st answer:

$2x^4 - 2x^3 - 12x^2$

or

$10x^4 - 10x^3 - 60x^2$

or

$3x^4 - 3x^3 - 18x^2$

etc.

$$(x-3)(x+2)x^2$$
$$= (x^2 - x - 6)x^2$$
$$= x^4 - x^3 - 6x^2$$

Name: ______________________________

10. Given that -1 is a solution, find all solutions, both real and complex, of the following equation: $x^3+x^2+5x+5=0$.

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 5 & 5 \\ & \downarrow & -1 & 0 & -5 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5}=\pm\sqrt{5}\,i$

$\boxed{\underline{\text{Solutions}}:\ x=-1,\ x=\pm\sqrt{5}\,i}$

11. Let $R(x)=x^3+3x^2-10x$. Then $R(3)=24$. For what other values of x is $R(x)=24$? To receive full credit, you must show all of your work.

3 is a zero of $x^3+3x^2-10x-24$.

So, using long division, we divide $x^3+3x^2-10x-24$ by $x-3$, and find a quadratic factor:

$$\begin{array}{r|l} & x^2+6x+8 \\ \hline x-3 & x^3+3x^2-10x-24 \\ & -(x^3-3x^2) \\ \hline & 6x^2-10x \\ & -(6x^2-18x) \\ \hline & 8x-24 \\ & (8x-24) \\ \hline & 0 \end{array}$$

Now, we find zeros of x^2+6x+8: $x^2+6x+8=0$

$(x+4)(x+2)=0$

$\boxed{x=-4,\ x=-2}$

*Could have used synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & 3 & -10 & -24 \\ & \downarrow & 3 & 18 & 24 \\ \hline & 1 & 6 & 8 & 0. \end{array}$$

Name: ______________________________

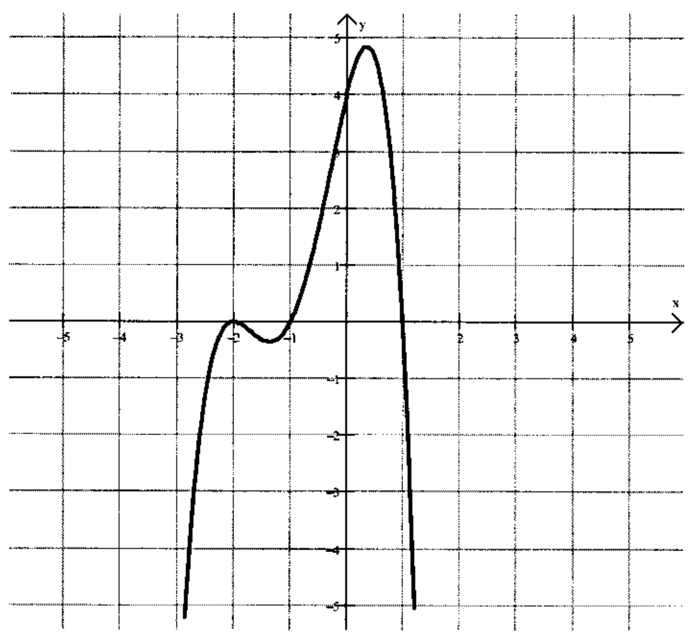
12. Given that -6 and -1 are roots of the following polynomial, find all other roots, real and complex, of the polynomial: $f(x) = x^4 + 7x^3 + 7x^2 + 7x + 6$.

$-6 \rfloor$	1	7	7	7	6
	$\downarrow$	-6	-6	-6	-6
$-1 \rfloor$	1	1	1	1	0
	$\downarrow$	-1	0	-1	
	1	0	1	0	

$\boxed{\text{Roots: } -6, -1, \pm i}$

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm i$$

13. Given the graph on the right, decide whether the following statements are **True or False**. You may assume nothing interesting happens outside the window shown.



a) This polynomial has a positive leading coefficient.

FALSE

b) The polynomial has a positive constant term.

TRUE

c) The polynomial does not have any repeated roots.

FALSE

d) The polynomial has odd degree.

FALSE

e) As x tends to both positive and negative infinity, the polynomial tends towards negative infinity.

TRUE

Name: ______________________________

14. What is the domain of the function $f(x) = 5 + 2\log(3x+2)$?

↑ must be positive

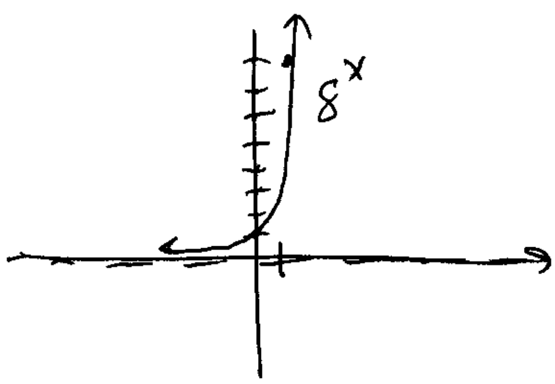
$3x + 2 > 0$

$3x > -2$

$$\boxed{x > -2/3}$$

15. What is the horizontal asymptote of the function $f(x) = 8^x + 3$?

$$\boxed{y = 3}$$

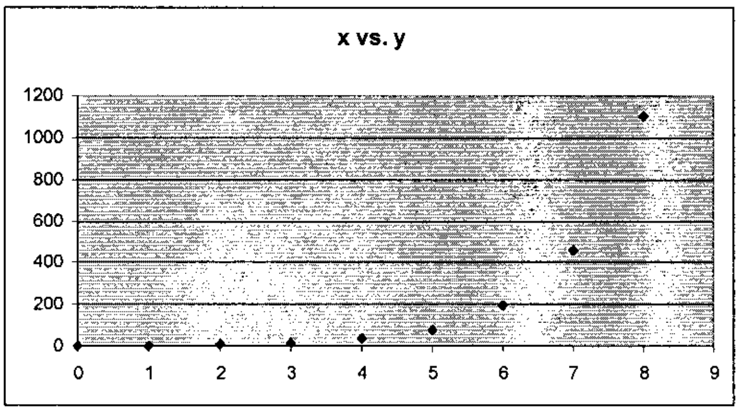


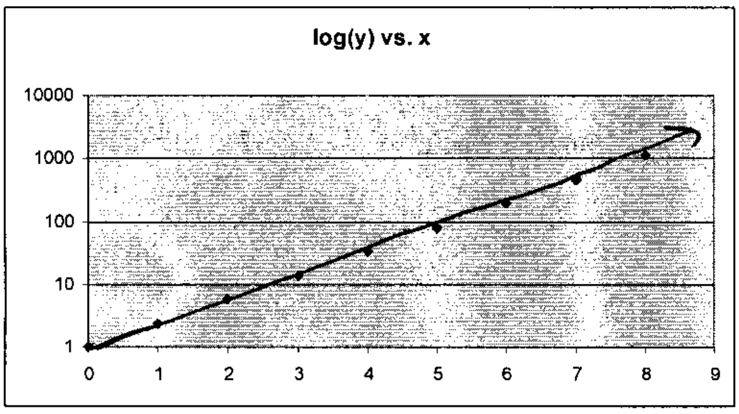
$y = 0$: Horizontal asymptote of $y = 8^x$

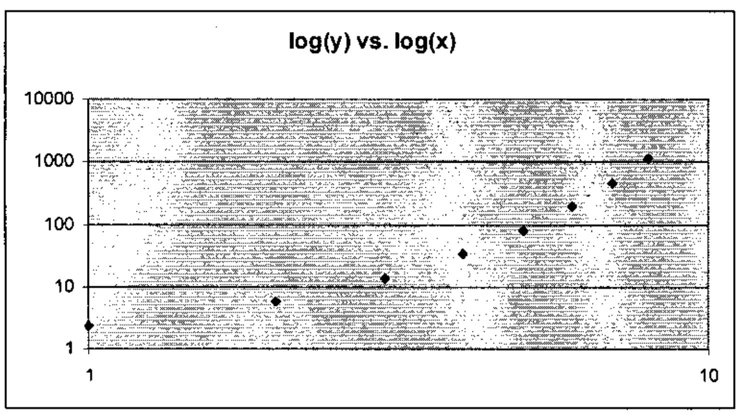
Shifting it up 3 units gives $y=3$ for the horizontal asymptote of $f(x) = 8^x + 3$.

Name: ______________________________

16. Some students have a data set, for which they create standard, log-log, and semi-log plots. (The plots are given below). One of the students claims that a power model would be an appropriate fit for the data set. Is the student's claim correct? Why or why not? ***You must explain yourself in complete sentences to receive full credit.***







The student is wrong.
Since the semilog plot (log y vs. x) gives a linear pattern, an exponential model would be appropriate.