Section 3.1 - Exponential Functions

Note Title 3/5/2008

$f(x) = a^x$

(a: some fixed number) "base"

(not to be confused with $g(x) = x^a$ ← power function)

Ex) $f(x) = 2^x$

$f(0) = 2^0 = 1$ $\quad (0, 1)$

$f(1) = 2^1 = 2$ $\quad (1, 2)$

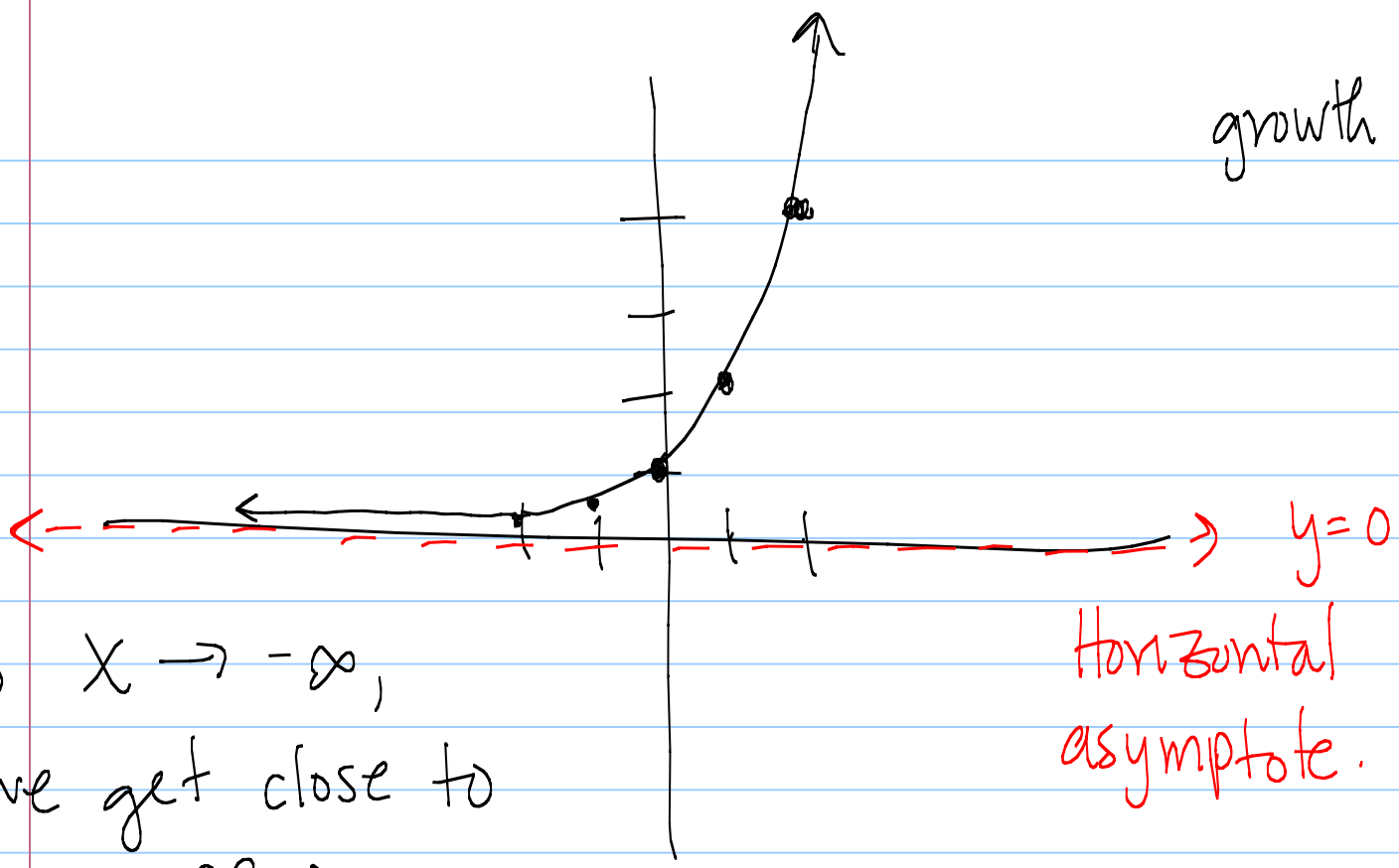
$f(2) = 2^2 = 4$ $\quad (2, 4)$

$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ $\quad (-1, \frac{1}{2})$

$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ $\quad (-2, \frac{1}{4})$

} Plot these

$f(-100) \longrightarrow$ output will be very close to zero



as $x \to -\infty$, we get close to zero

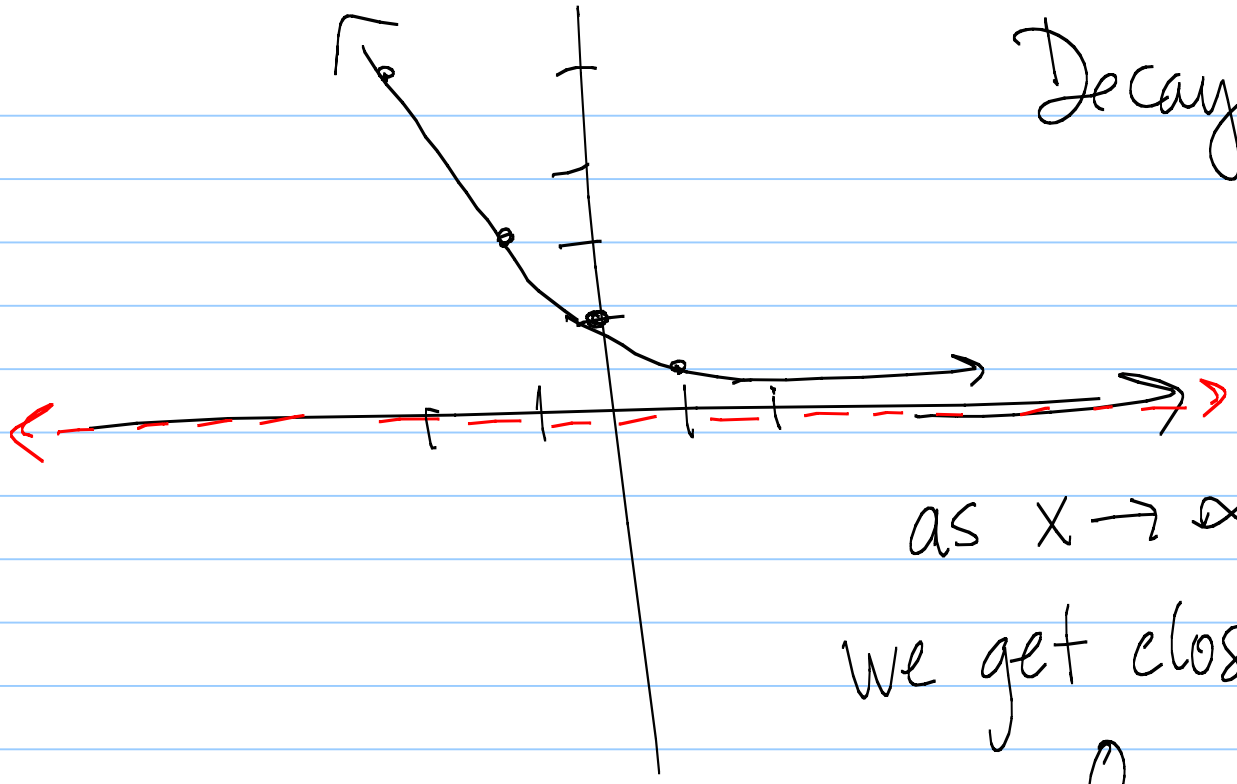
Horizontal asymptote.

Ex) $f(x) = 2^{-x}$

$f(-2) = 2^{-(-2)} = 2^2 = 4$

$f(-1) = 2^{-(-1)} = 2^1 = 2$

x	f(x)
-2	4
-1	2
0	1
1	1/2
2	1/4



Decay

as $x \to \infty$

we get close to

0

Ex) Suppose $f(x) = 6^{-3x} + 4$

$$f(0) = 6^{-3(0)} + 4 = 6^0 + 4$$

$$= 1 + 4$$

$$= 5$$

$$f(4/5) = 6^{-3\left(\frac{4}{5}\right)} + 4$$

$$\approx 4.0136$$

Horizontal asymptote

$y = 4$

$g(x) = 6^{-3x}$

$y = 0$ (Horiz. asymptote)

Ex2) (from online)

$$f(x) = 3^{-3x} + 5$$

$$f(0) = 3^0 + 5 = 1 + 5 = 6$$

$$f(-1/2) = 3^{(-3 \cdot \frac{-1}{2})} + 5$$

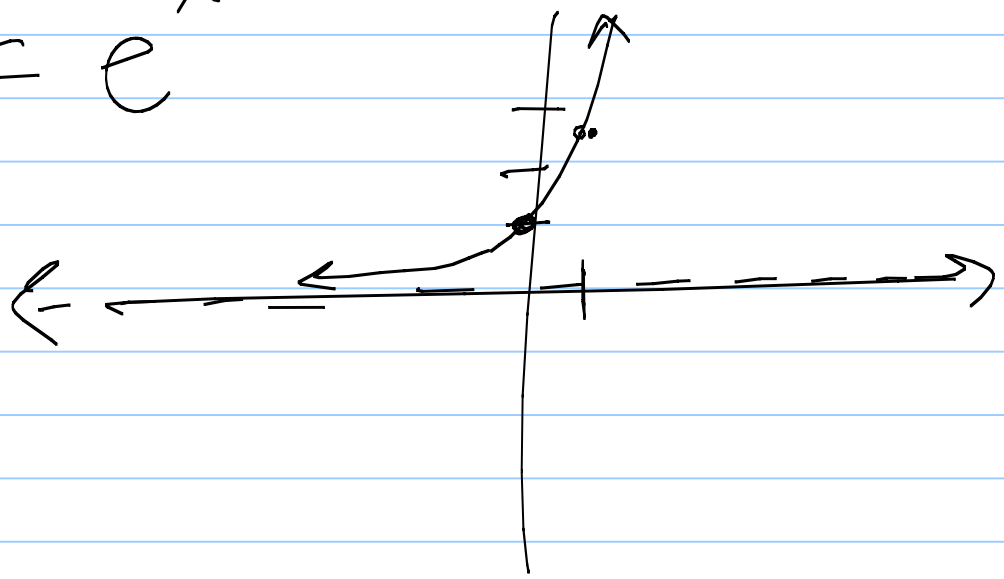
$$= 3^{3/2} + 5$$

$$= 10.1962$$

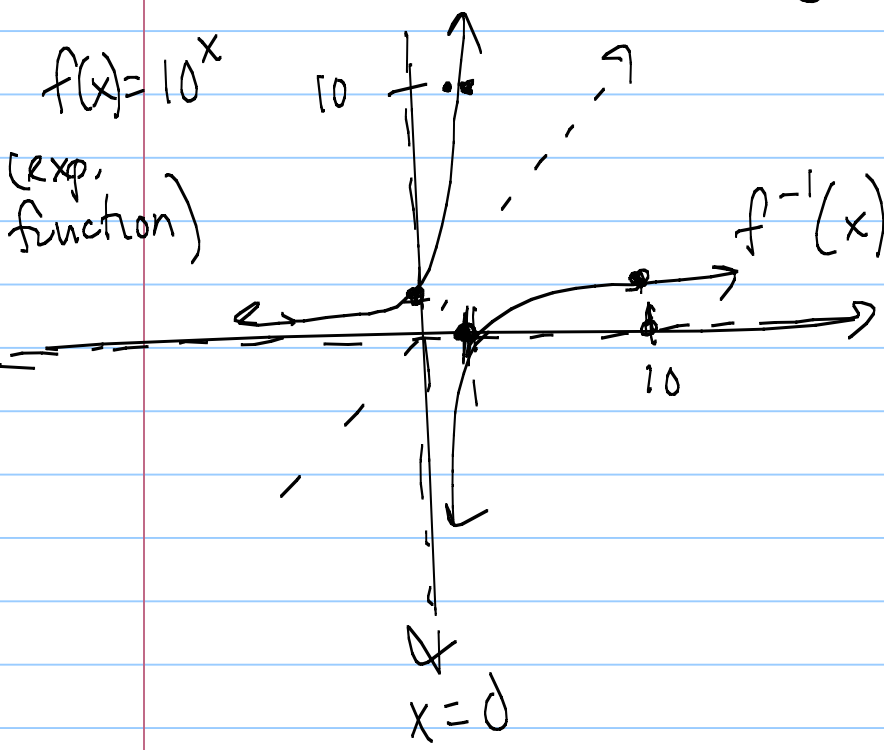
A special number: "Euler Number"

$$e \approx 2.718281828\ldots$$

$$f(x) = e^x$$



Section 3.2: Logarithmic Functions



$f^{-1}(x) = \log(x)$ → only positive values of x.

Domain of $\log(x)$: $x > 0$

$\log(x)$ → "common log"

$\ln(x)$ → "natural log"

} Domain: $x > 0$

* e^x & $\ln x$ are inverses of each other.

Ex) Find domain of $f(x) = 2\ln(x-1)$.

$f(x) = 2\ln(x-1)$

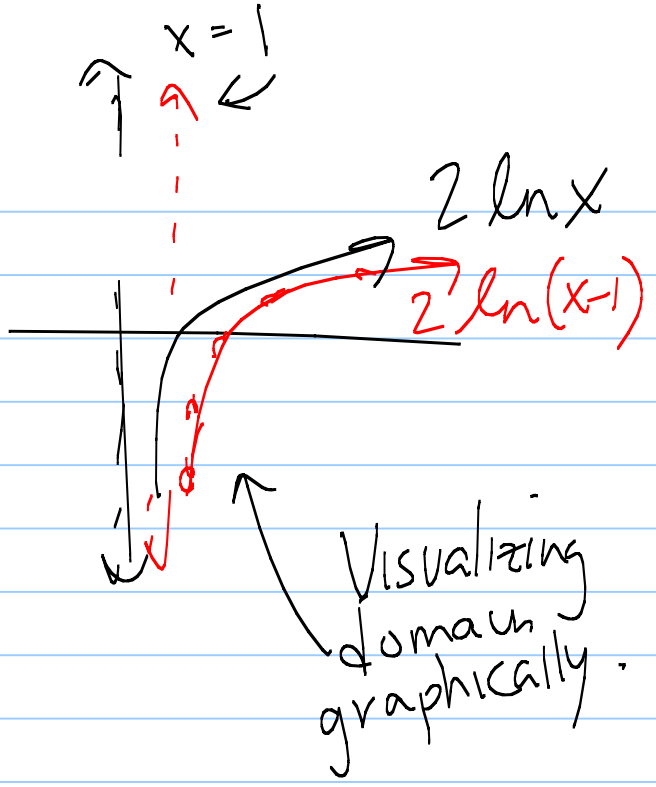
$(x-1)$ must be positive.

Finding domain algebraically:

$(x-1) > 0$

$+1 \quad +1$

$\boxed{x > 1}$.



How to find function values:

Compute $f(10)$; $f(x) = 2\ln(x-1)$

$f(10) = 2\ln(10-1)$

$= 2\ln(9)$

$= 4.3944$

Ex) Online HW

Find domain: $f(x) = -4\log(-x+4)$

The quantity $-x+4$ has to be positive.

$$-x+4 > 0$$

$$+x \qquad +x$$

$$4 > x$$

$x < 4$ Domain