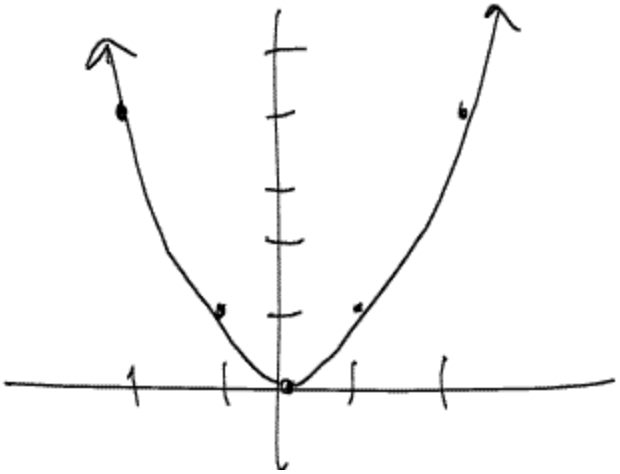
Section 2.1 & 2.2 / Quadratic functions

$f(x) = x^2$

x	-2	-1	0	1	2
f(x)	4	1	0	1	4



Tools:

① Multiplying Binomials:

$(x+5)(x-2)$

F: first
O: outer
I: Inner
L: last

$x^2 - 2x + 5x - 10$

$= \underline{x^2 + 3x - 10}$

② Factoring

Ex) $x^2 + 6x + 8$

$(x+2)(x+4)$

Check. $x^2 + \underbrace{4x + 2x}_{6x} + 8$

③ Zero-Product Property

If $AB = 0$, then either

$A = 0$ or $B = 0$.

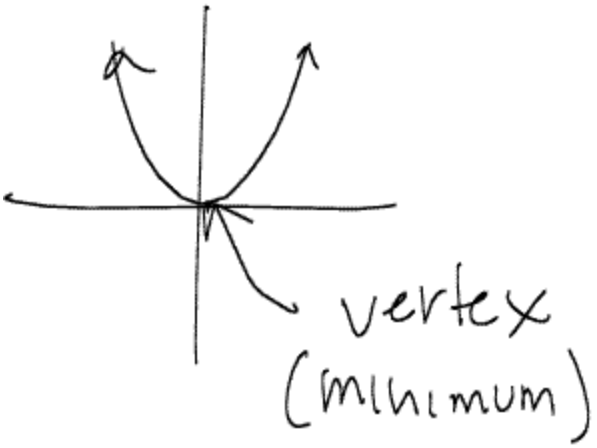
Ex) $\underbrace{(x+2)}_{A}\underbrace{(x+4)}_{B} = 0$

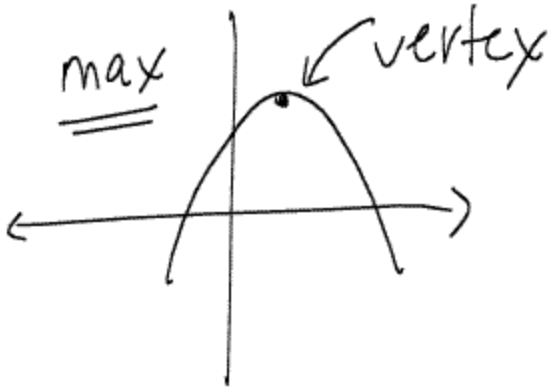
Either $x+2=0$ or $x+4=0$

$\quad -2 \quad -2$ or $\quad -4 \quad -4$

$x = -2$ or $x = -4$

① Vertex - Form of a Quadratic Function





Vertex form: $f(x) = a(x-h)^2 + k$

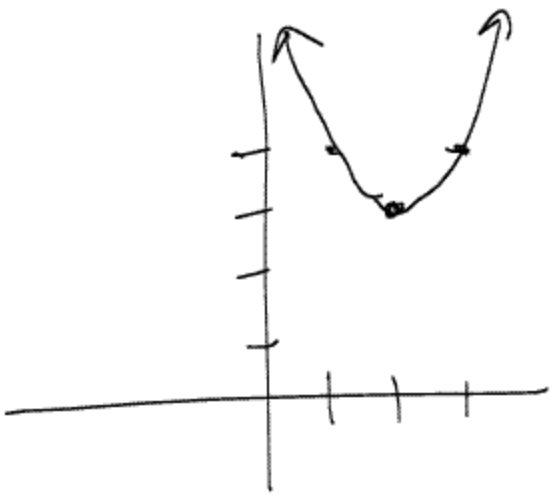
(h, k): vertex

Ex) $f(x) = (x-2)^2 + 3$, Y1: $(x-2)^2+3$

$a = 1$

vertex: $(2, 3)$

$a > 0$ means that ↗ parabola opens upward



② Standard form: $f(x) = ax^2 + bx + c$

(a, b, c are real #s)

Vertex form

$(x-2)^2 + 3$

$(x-2)(x-2) + 3$

$x^2 - 4x + 4 + 3$

Standard Form

$x^2 - 4x + 7$

$a = 1$

$b = -4$

$c = 7$

Example: (like Online HW)

Complete the Square: $x^2 + 5x - 7$

$a = 1$

Vertex form:

$a(x-h)^2 + k$

$(x-h)^2 + k$

$= (x-h)(x-h) + k$

$= x^2 - 2hx + h^2 + k$

↑ constant term

$x^2 + 5x - 7$

$\frac{-2h}{-2} = \frac{5}{-2}$

$h = -5/2$

$h^2 + k = -7$

$\left(\frac{-5}{2}\right)^2 + k = -7$

$\frac{25}{4} + k = -7$

$\frac{-25}{4} \qquad \frac{-25}{4}$

$k = \frac{-28}{4} - \frac{25}{4}$

$k = -\frac{53}{4}$

Vertex: (h, k)

$\left(-\frac{5}{2}, -\frac{53}{4}\right)$

$\left(x - (-5/2)\right)^2 - 53/4$

Formula for finding the vertex: (given a parabola in std. form)

$f(x) = ax^2 + bx + c$

$h = \frac{-b}{2a}$

$k = f(h)$

Ex) Find the vertex:

$f(x) = 2x^2 - 3x + 1$

$a = 2$

$b = -3$

$h = \frac{-(-3)}{2(2)}$

To find

$h = \frac{3}{4}$

$k = f\left(\frac{3}{4}\right)$

$= 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$

$= 2\left(\frac{9}{16}\right) - \frac{9}{4} + 1 =$ you finish

Ex) Given a parabola with vertex at $(-1, 4)$, passing through $(1, -5)$. Find the equation of the parabola in standard form.

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x+1)^2 + 4$$

$$-5 = a(1+1)^2 + 4$$ Solve for a

$$-5 = a(2)^2 + 4$$

$$-5 = 4a + 4$$

$$-4 \qquad -4$$

$$\frac{-9}{4} = \frac{4a}{4}$$

$a = -9/4$

$$f(x) = -\frac{9}{4}(x+1)^2 + 4$$ (vertex form)

multiply

$$= -\frac{9}{4}(x+1)(x+1) + 4$$

$$= -\frac{9}{4}\left(x^2+2x+1\right)+4$$

$$= -\frac{9}{4}x^2 - \frac{18}{4}x - \frac{9}{4} + 4$$

$$= -\frac{9}{4}x^2 - \frac{9}{2}x - \frac{9}{4} + \frac{16}{4}$$

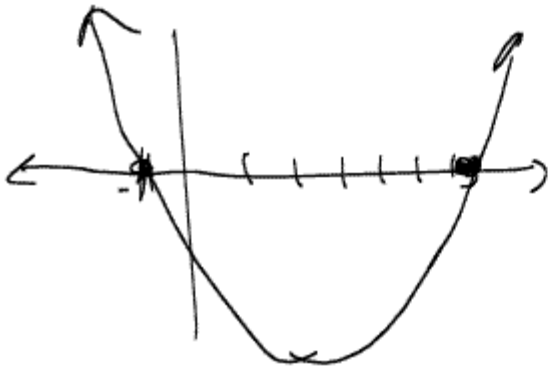
$$= -\frac{9}{4}x^2 - \frac{9}{2}x + \frac{7}{4}$$ (standard form)

Solving Quadratic Equations

① Method 1: Use Factoring

Solve for x. $x^2 - 5x - 6 = 0$

Find the "roots"



$(x-6)(x+1) = 0$

$x - 6 = 0$ or

$x = 6$

or $x + 1 = 6$

$x = -1$

Method 2: Quadratic formula

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Ex) Find the roots (solve for x)

$$2x^2+24x+24$$

$a=2$
$b=24$
$c=24$

$$=\frac{-24\pm\sqrt{24^2-4(2)(24)}}{2(2)}$$

$$=\frac{-24\pm\sqrt{576-192}}{4}$$

$$=\frac{-24\pm\sqrt{384}}{4}$$

$$= \frac{-24}{4} \pm \frac{\sqrt{384}}{4}$$

$\sqrt{384}$

$2 \mid 384$

$2 \mid 192$

$2 \mid 96$

$2 \mid 48$

$2 \mid 24$

$2 \mid 12$

$2 \mid 6$

3

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

$$-6 \pm \frac{8\sqrt{6}}{4}$$

$$= \boxed{-6 \pm 2\sqrt{6}}$$