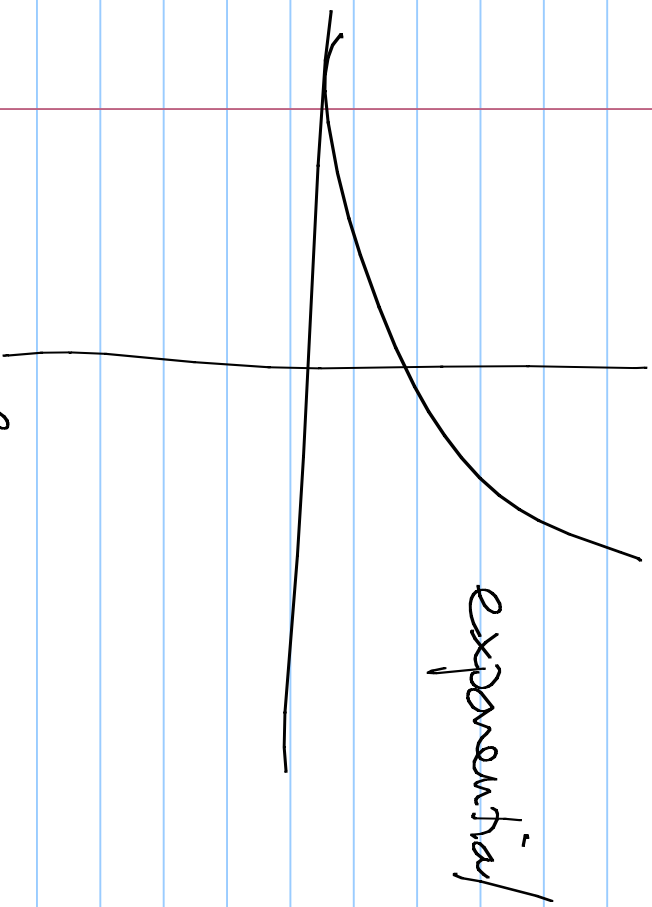


# Properties of Logarithms

Note Title

10/21/2009



inverse is a  
logarithm

inverse of  
 $10^x$   
is  $\log(x)$   
"common log"

inverse of  
 $e^x$   
is  $\ln(x)$   
"natural log"

# Properties of Exponents have matching properties of logs

## Exponents

$$10^a \cdot 10^b = 10^{a+b}$$

$$\frac{10^a}{10^b} = 10^{a-b}$$

$$(10^a)^b = 10^{a \cdot b}$$

## Log

$$\log(ab) = \log(a) + \log(b)$$

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$

$$\log(a^b) = b \log(a)$$

## Ln

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^b) = b \ln(a)$$

$$\log(10) = 1$$

$$\ln(e) = 1$$

$$\log(10^x) = 10^{\log(x)} = x$$

$$\ln(e^x) = e^{\ln(x)} = x$$

Examples  $\log(7a) = \log(7) + \log(a)$

$$\log(x^3) = 3 \log(x)$$

$\log(x^2 + 1)$  doesn't simplify  
 ~~$\log(a+b)$~~  STOP

Online  $\ln(a) = 0.858$ ,  $\ln(b) = 2.826$

$$\ln(2) = 0.693, \ln(3) = 1.099$$

$$\ln(4a^6) = \ln 4 + \ln(a^6) = 2\ln(2) + 6\ln(a)$$

$6 \cdot 534$  ←

$$= 2(0.693) + 6(0.858)$$

$$\ln \left( \sqrt[3]{a/b^2} \right) = \frac{1}{3} \ln (a/b^2)$$

$$\sqrt[3]{X} = X^{1/3} = \frac{1}{3} \left[ \ln(a) - \ln(b^2) \right]$$

$$= \frac{1}{3} \left[ \ln(a) - 2 \ln(b) \right]$$

$$= \frac{1}{3} \left[ 6.858 - 2(2.826) \right]$$

$$= \underline{\underline{-1.598}}$$

## Equations

Isolate the exponential or logarithm  
Undo with log or exponent

Then solve:

Example

$$e^{7x+4} = 5$$

$$\ln(e^{7x+4}) = \ln 5$$

$$7x+4 = \ln 5$$

$$7x = \ln 5 - 4$$

$$x = \frac{\ln 5 - 4}{7}$$

Example

$$-7 \ln(x) + 5 = -2$$

$$\frac{-7 \ln(x)}{-7} = \frac{-7}{-7}$$

$$e^{\ln(x)} = 1$$

$$x = e^1 = e = 2.718$$

NOTE

exp(x) MEANS

$e^x$

Excel

=exp(1)

$$\boxed{3^x - 7 = 2} \\ +7 \quad +7$$

$$3^x = 9$$

$$\log(3^x) = \log(9)$$

$$x \log(3) = \log(9)$$

$$x = \frac{\log(9)}{\log(3)} = 2$$

FV PV

Future Value vs. Present Value

$$F.V. = P.V. \left(1 + \frac{r}{k}\right)^{kn}$$

$r$  Interest rate  
 $k$  compoundings per year

$$F.V. \left(1 + \frac{r}{k}\right)^{-n} = P.V.$$

$n$  # compounding periods

\$65,000 at 13% compounded quarterly

today worst future

$$F.V. = 65000 \left(1 + \frac{.13}{4}\right)^{10} = 89,498.13$$

$10$  quarters

What is Future Value in 11 years?

$$F.V. = 65000 \left(1 + \frac{.13}{4}\right)^{44}$$

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$$F.V. = \$77000$$

16% compounded semi annually  
in 25 years

$$P.V. = 77000 \left(1 + \frac{.16}{2}\right)^{-50} = \$1641.73$$