

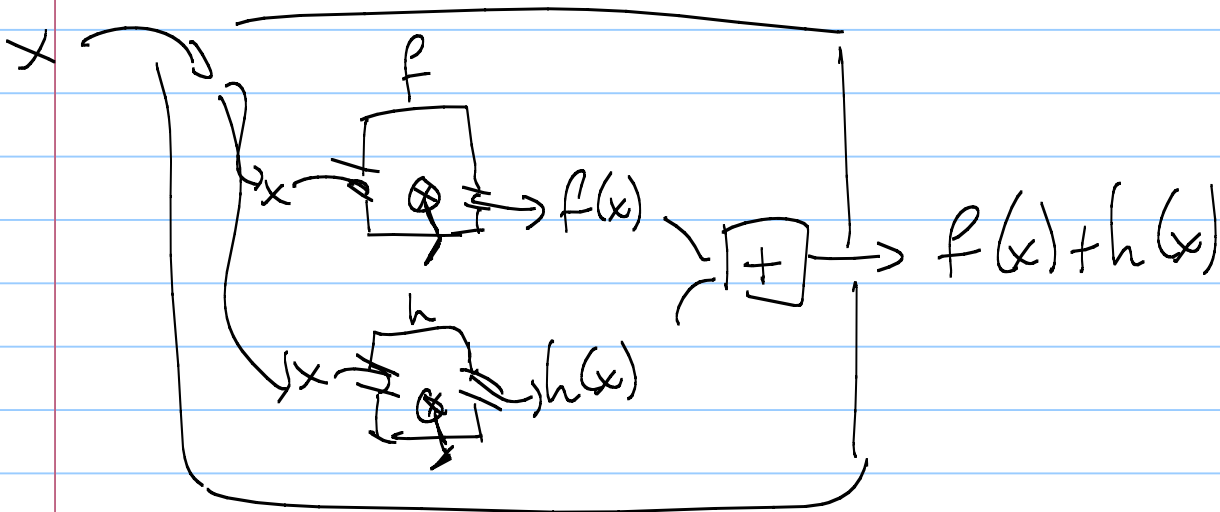
Building New Functions From Old

Note Title

9/23/2009

Sections 4.2 and 4.3

Adding Functions $f(x) + h(x)$



$$f(x) = 2x^2 - 3x + 4$$

$$h(x) = 3x + 1$$

$$g(x) = 4x^2 - 6x + 1$$

$$f+h(x)$$

$$= 2x^2 - 3x + 4$$

$$+ 3x + 1$$

$$\hline 2x^2 + 5$$

$$g-h \quad 4x^2 - 6x + 1$$

$$- 3x + 1$$

$$\hline 4x^2 - 9x$$

$$\boxed{4x^2 - 9x}$$

$$g(x) \cdot h(x) = (4x^2 - 6x + 1)(3x + 1)$$

distributive
law \rightarrow

$$= (4x^2 - 6x + 1)3x + (4x^2 - 6x + 1) \cdot 1$$

$$= 4x^2 \cdot 3x - 6x \cdot 3x + 1 \cdot 3x + 4x^2 \cdot 1 - 6x \cdot 1 + 1 \cdot 1$$

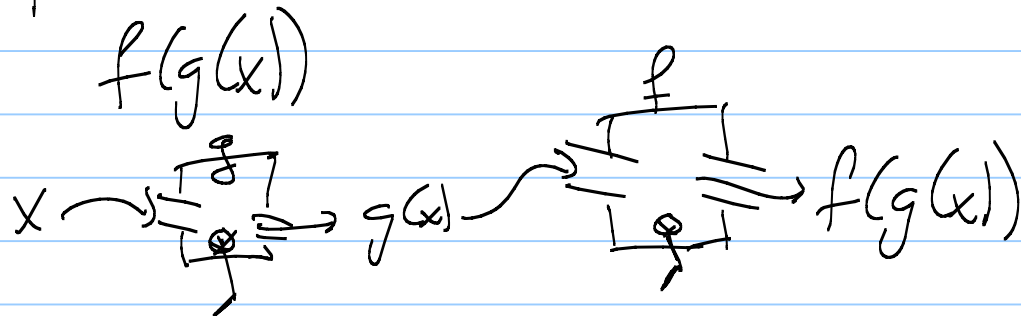
This is correct, but there's
a better way to organize
this

$$\begin{array}{r|l}
 4x^2 - 6x + 1 & \\
 \hline
 12x^3 & -18x^2 & 3x & 3x \\
 \hline
 4x^2 & -6x & 1 & +1
 \end{array}$$

$$12x^3 - 18x^2 - 3x + 1$$

$$\boxed{12x^3 - 14x^2 - 3x + 1}$$

Composition



Recall

$$h(x) = 3x + 1$$

$$g(x) = 4x^2 - 6x + 1$$

$$h(g(x)) = 3g(x) + 1$$

$$= 3(4x^2 - 6x + 1) + 1$$

$$= 12x^2 - 18x + 3 + 1$$

$$= 12x^2 - 18x + 4$$

$$g(h(x)) = 4(h(x)^2) - 6h(x) + 1$$

$$= 4(3x+1)^2 - 6(3x+1) + 1$$

$$(3x+1)^2$$

3x+1		
9x ²	3x	3x
3x	1	+1

$$9x^2 + 6x + 1$$

$$= 4(9x^2 + 6x + 1) - 6(3x + 1) + 1$$

$$= 36x^2 + 24x + 4 - 18x - 6 + 1$$

$$= 36x^2 + 6x - 1$$

$$\boxed{36x^2 + 6x - 1}$$

Why combine functions?

Know Revenue

Know Cost

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Fixed Costs Costs No Matter How
Much You Have

Variable Cost Costs that grow (or
on occasion shrink)
as the number changes

Apartment Building A+ \$400
all 100 units rented

How much Revenue For every \$20 rent ↑
1 fewer apartment will
be rented

Rent
 $(400 + 20x)$

Apts

$(100 - x)$

$(400 + 20x)(100 - x)$

Costs \$1000

Var. Costs \$5 per apartment

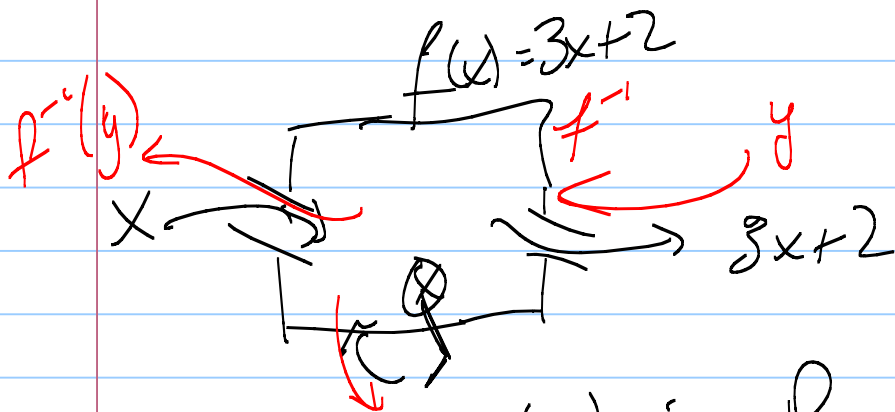
$$\text{Revenue} = (20x + 400)(100 - x) \quad A(x) = 100 - x$$

$$\text{Costs} = 1000 + 5A \quad A = \# \text{ apartments}$$

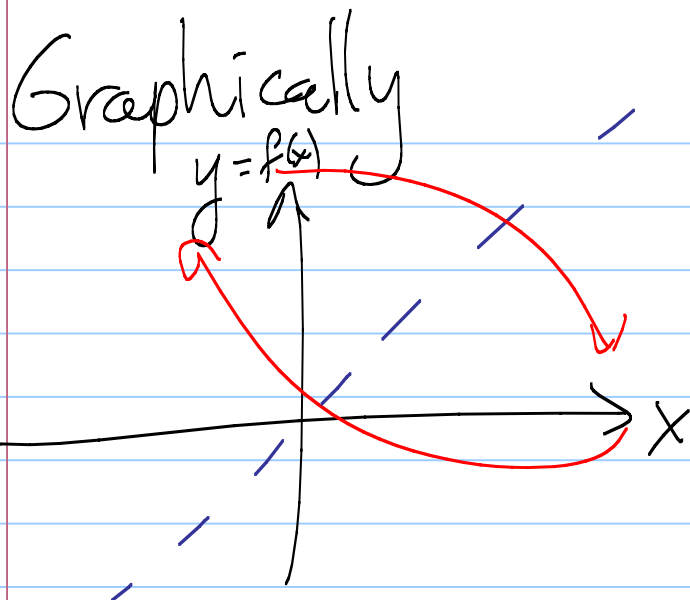
$$1000 + 5(100 - x)$$

$$(20x + 400)(100 - x) - (5(100 - x) + 1000)$$

Inverse Functions



$f^{-1}(x)$ is the same machine
RUN BACKWARDS



switch the x and
y that gives
the inverse

For formulas

$$y = 3x + 1$$

Switch x, y \rightarrow $x = 3y + 1$

Solve for y

$$x - 1 = 3y$$

$$f^{-1}(x) = (x - 1) / 3$$

$$\frac{x - 1}{3} = y$$