# Roots of Polynomials

Note Title 11/4/2009

Solve $x^3 + 5x^2 + 2x - 8 = 0$

Idea
Factor, then use Zero Product Property

Guess $x=1$ $1^3 + 5\cdot 1^2 + 2\cdot 1 - 8$

$= 1 + 5 + 2 - 8$

$= 0$ ✓

This being a problem fall back on Guess & Check

$x=1$ works

## ROOT-FACTOR THEOREM

We know that if our polynomial has a factor $(x-3)p(x) = 0$

then $x-3=0$ OR $p(x)=0$

$x=3$ OR . . . .

THIS WORKS IN REVERSE!

~~$(x-1=0)$~~

If $x=1$ is a root of $x^3+5x^2+2x-8$

then $x^3+5x^2+2x-8=(x-1)p(x)$

Given a factor I can divide it out

$$x-1 \overline{)\, x^3+5x^2+2x-8}$$

$$x^2+6x+8$$

$$-(x^3-x^2)$$

$$6x^2+2x$$

$$-(6x^2-6x)$$

$$8x-8$$

$$8x-8$$

$$0$$

5th Grade

$$13 \overline{)\, 716}$$

$$55$$

$$65$$

$$66$$

$$65$$

$$1 \leftarrow \text{remainder}$$

$$x^3+5x^2+2x-8=(x-1)(x^2+6x+8)=0$$

$x-1=0$ OR $x^2+6x+8=0$

$x=1$

$(x+4)(x+2)=0$

$x+4=0$ OR $x+2=0$

$x=-4$ $x=-2$

# Synthetic Division

$x-1$ $\qquad x^3+5x^2+2x-8$

root $\qquad$ coefficients (must include 0's)

1 | 1 5 2 −8

| | 1 | 6 | 8 |
|---|---|---|---|
| 1 | 1 | 6 | 8 |
| | 1 | 6 | 8 | 0

Go down by addition
Go up & over by multiplication

$x^2+6x+8$ remainder

## Online Example $\qquad x^3+5x^2+11x+15=0$

$x=-3$ is one root

−3 | 1 5 11 15

| | 1 | 2 | 5 |
|---|---|---|---|
| −3 | −3 | −6 | −15 |
| | 1 | 2 | 5 | 0

$x^3+5x^2+11x+15=$

$(x+3)(x^2+2x+5)=0$

$x+3=0$

$x=-3$

↓ Quadratic Formula

$$\frac{-2\pm\sqrt{4-4\cdot1\cdot5}}{2\cdot1}$$

$$\frac{-2\pm\sqrt{-16}}{2}=\frac{-2\pm4i}{2}$$

$-1\pm2i$

$\sqrt{-1}=i$

$x^3 - 8x^2 + 22x - 20 = 0$ $\quad x = 2$

$$\begin{array}{c|cccc} 2 & 1 & -8 & 22 & -20 \\ & & 2 & -12 & 20 \\ \hline & 1 & -6 & 10 & 0 \end{array} \checkmark$$

$(x-2)(x^2-6x+10) = 0$

$x-2=0$ OR $x^2-6x+10=0$

$x=2$ $\qquad \dfrac{6 \pm \sqrt{36-4\cdot1\cdot10}}{2}$

$\dfrac{6 \pm \sqrt{-4}}{2}$ $\qquad \dfrac{6 \pm 2i}{2}$

$3 \pm i$

(d)

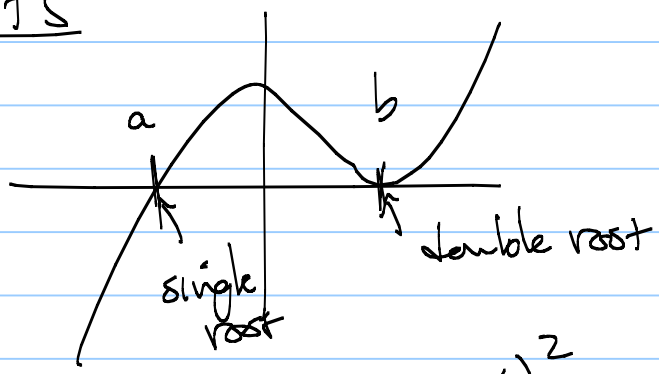
# Remainders

$$f(x) = x^3 + 3x^2 + 7x + 1 = 0$$

I think 1 is a root

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 3 | 7 | 1 |
| | | 1 | 4 | 11 |
| | 1 | 4 | 11 | 12 ← Remainder = $f(1)$ |

## Double Roots




$(x-a)$ $(x-b)^2$

**Online** double roots at 2, −2

$$(x-2)^2 \cdot (x+2)^2$$

$$(x-2)^2 = x^2 - 4x + 4$$

$$(x+2)^2 = x^2 + 4x + 4$$

$$(x-2)^2(x+2)^2 = x^2 - 4x + 4$$

| | | | |
|---|---|---|---|
| $x^4$ | $-4x^3$ | $4x^2$ | $x^2$ |
| $4x^3$ | $-16x^2$ | $16x$ | $+4x$ |
| $4x^2$ | $-16x$ | $16$ | $+4$ |