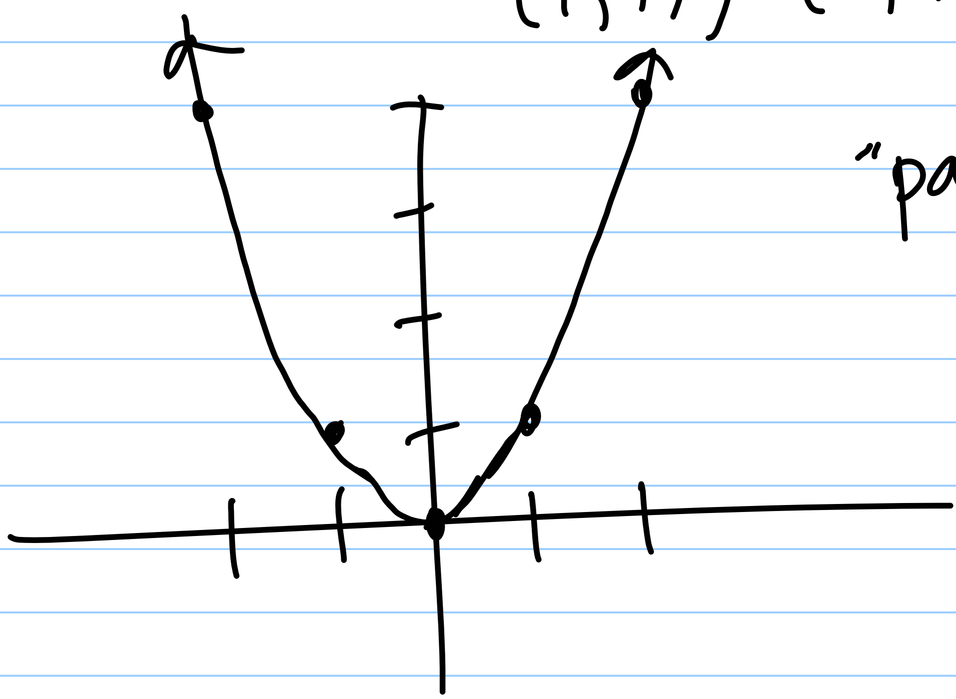
Section 3.1 & 3.2: Quadratic Functions

Note Title 9/16/2009

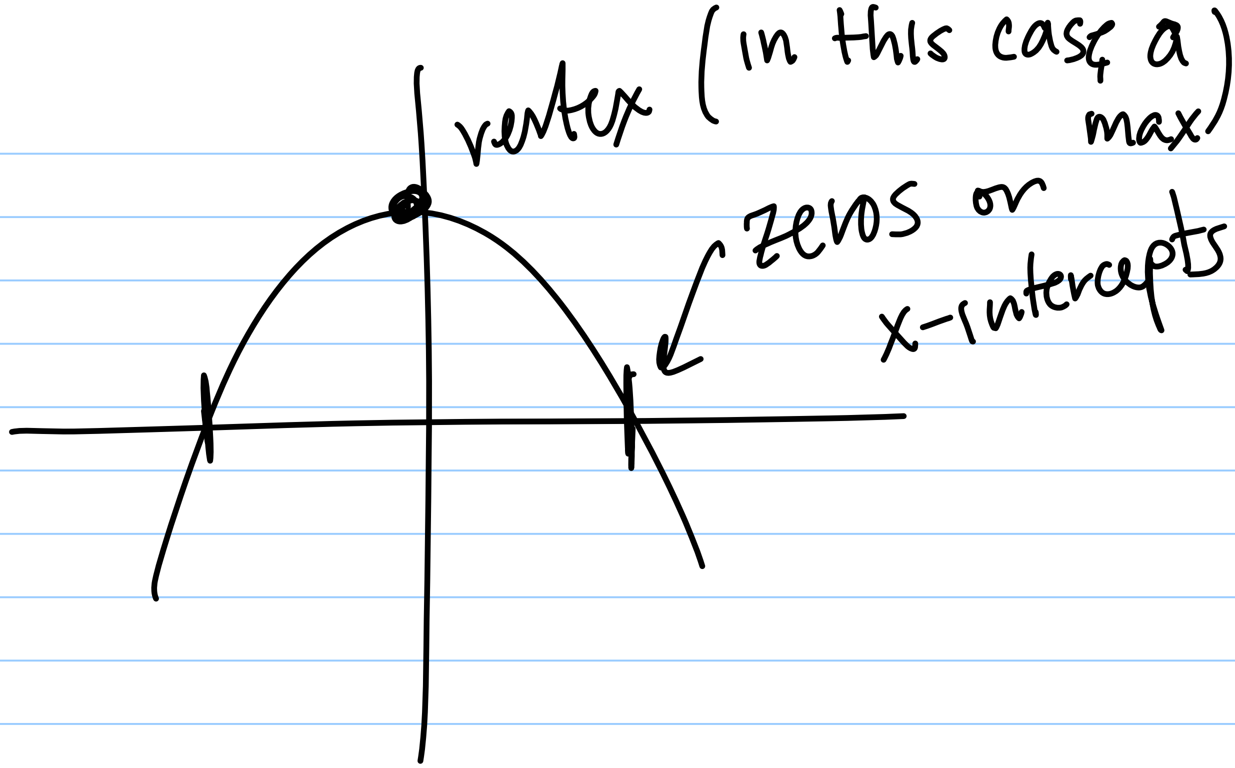
$f(x) = x^2$

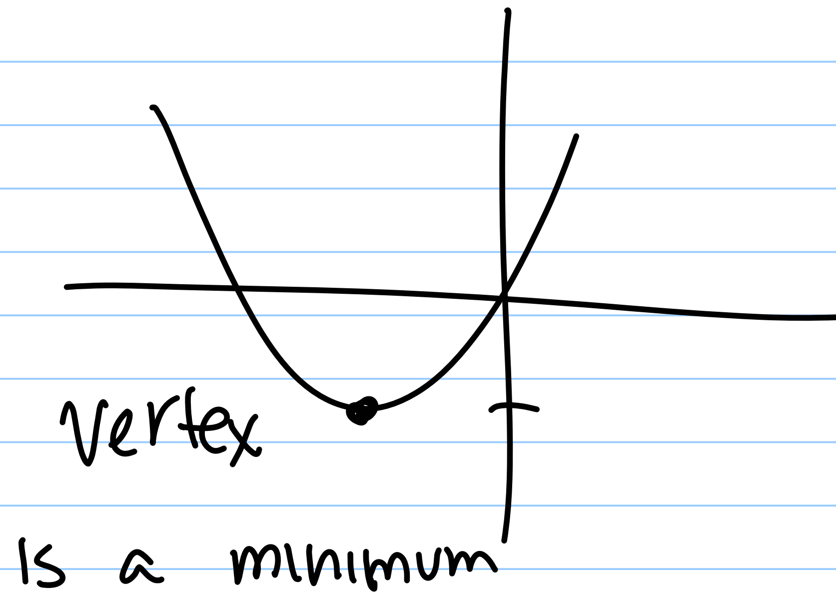
x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

Ordered pairs: $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$



"parabola"


vertex (in this case a max)
zeros or x-intercepts


vertex
is a minimum

② Vertex Form of a Parabola

$$f(x) = a(\underline{x-h})^2 + K$$

(h, k): location of the vertex.

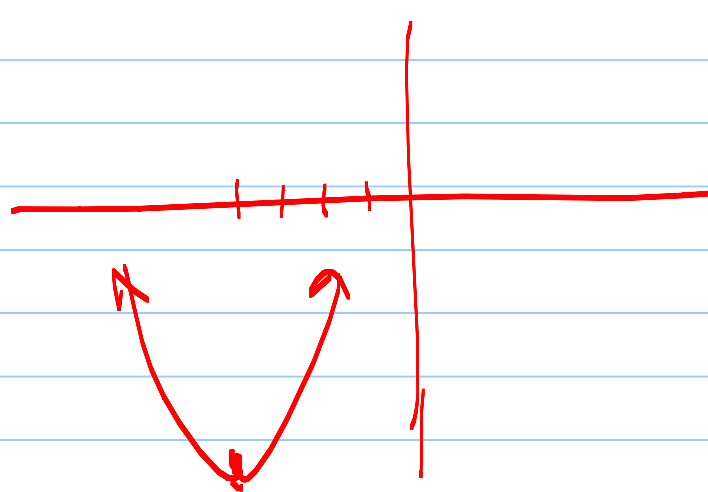
$a > 0$: parabola opens upward

$\underline{a < 0}$: parabola opens downward

Ex) $f(x) = (\underline{x+4})^2 - 25$

$a = 1$

$(h, k) = (-4, -25)$ Vertex



Standard form of a quadratic:

$$f(x) = ax^2 + bx + c$$

[Ex] $f(x) = x^2 + 8x - 9$

$a = 1$
$b = 8$
$c = -9$

[Ex] Put $f(x) = x^2 + 8x - 9$ in vertex form

* Use the process of "completing the square."
* This appears in the online hw

$$x^2 + 8x - 9 = x^2 + 8x \underbrace{+ 0}\, - 9$$

* $\frac{1}{2}$ of b

$$= \underbrace{x^2 + 8x + 16}\, \underbrace{-16 - 9}$$

$\frac{1}{2}(8) = 4$

$$= (x+4)(x+4) - 25$$

$4^2 = \underline{\underline{16}}$

$$= \boxed{(x+4)^2 - 25}$$

(4) Some other useful formulas:

Given $f(x) = ax^2 + bx + c$

the vertex is

$$h = \frac{-b}{2a} \quad \text{and} \quad k = f(h)$$

Ex) Find the vertex of

$f(x) = -x^2 + 4$, and graph

$f(x)$ on the xy plane.

$a = -1$

$b = 0$

$$h = \frac{-b}{2a} = \frac{-0}{2(-1)}$$

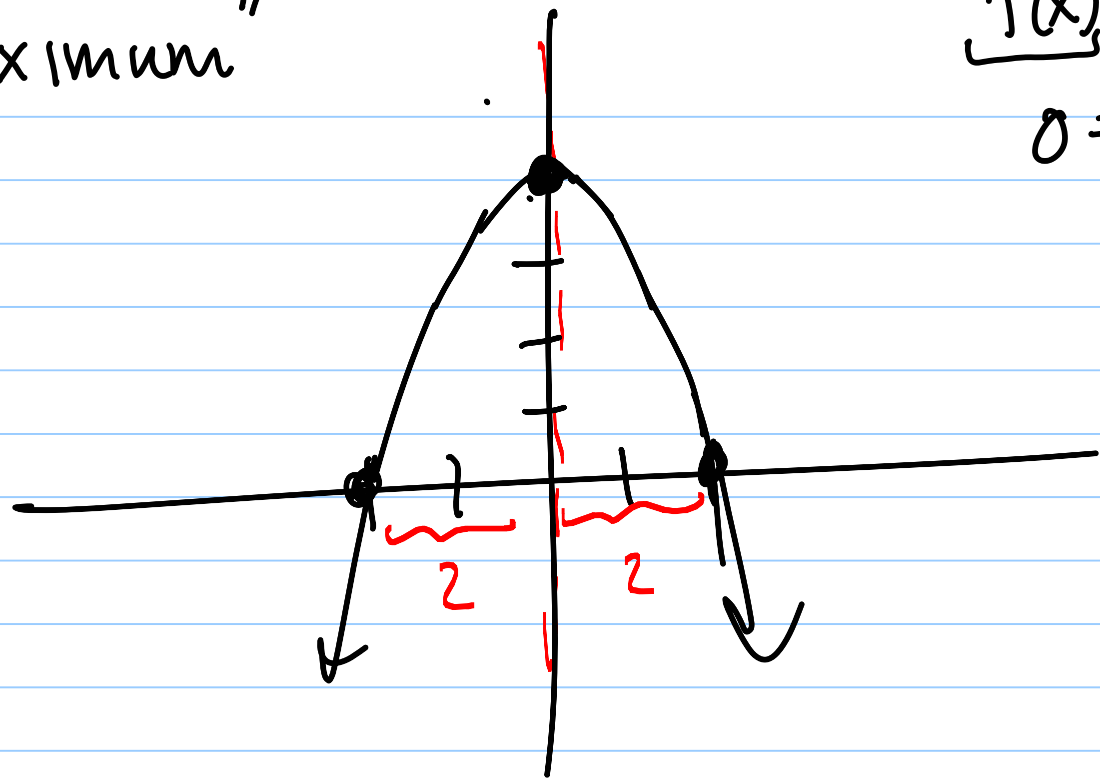
$h = 0$

$k = f(0) = 4$

vertex:

$(0, 4)$

"Maximum"



$f(x) = -x^2 + 4$

$0 = -x^2 + 4$

$x^2 = 4$

$x = 2$

-2

Figure out where $f(x)$ crosses the x-axis.

↳ Finding the x-intercepts

↳ Setting $y = 0$, and solving for x.

IC #1

$$R(x) = 270x - 90x^2$$

$x =$ # of lumens

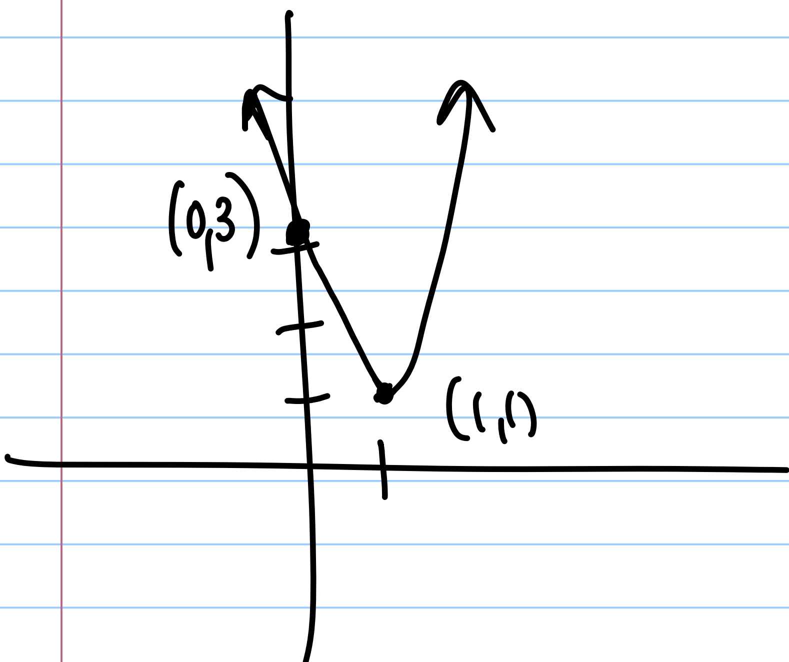
$R =$ rate of photo synth.

Vertex: $\frac{-b}{2a}$

$$= \frac{-270}{2(-90)}$$

$$= \frac{270}{180} = \boxed{3/2 \text{ lumens}}$$

More Examples on Parabolas



Find the equation of the parabola with vertex at (1,1) passing through (0,3)

Write answer in standard form:

$$ax^2+bx+c.$$

① Start with vertex form

(0, 3)

x f(x)

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-1)^2 + 1$$

$$3 = a(0-1)^2 + 1$$

② Find a

$$3 = a + 1$$

$\underline{a = 2}$

(3) Plug back a, h, & k into vertex form

$$f(x) = 2(x-1)^2 + 1$$

$(x-h)^2 + k$

vertex

(1, 1)

(h, k)

(4) Multiply out righthand side

$$f(x) = 2\left[(x-1)(x-1)\right] + 1$$

$$= 2\left[x^2 - 2x + 1\right] + 1$$

$$= 2x^2 - 4x + 2 + 1$$

$$\boxed{f(x) = 2x^2 - 4x + 3}$$

Good problem

Solving Quadratic Equations

① Factoring:

Solve $\underbrace{x^2 - 5x - 6}$ $= 0$

$$\underbrace{(x-6)}\underbrace{(x+1)} = 0$$

So, $x-6=0$ or $x+1=0$

"zero product property"

$$\boxed{x=6 \text{ or } x=-1}$$

2 Quadratic Formula

If $ax^2+bx+c=0$, the solutions are $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Solve: $x^2+8x+12=0$

$a=1$

$b=8$

$c=12$

$$\frac{-8\pm\sqrt{64-4(1)(12)}}{2(1)}$$

$$=\frac{-8\pm\sqrt{16}}{2}=\frac{-8\pm4}{2}$$

$$\frac{-8+4}{2} \quad \text{or} \quad \frac{-8-4}{2}=-6$$

$$=-2$$

$\boxed{\begin{array}{l}\underline{\text{Solutions}}\\ x=-2\\ \text{or } x=-6\end{array}}$

$$x^2 + 4x + 2 = 0$$

$a = 1$

$b = 4$

$c = 2$

$$\frac{-4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{8}}{2}$$

$\sqrt{8} = \sqrt{4 \cdot 2}$

$= 2\sqrt{2}$

$$= \frac{-4 \pm 2\sqrt{2}}{2}$$

$$= \boxed{-2 \pm \sqrt{2}}$$