Name: Key

Recitation Instructor and Time: _____

Studio College Algebra – Exam 3
April 7, 2009

1. Rewrite the formula $y^3 = \frac{z}{7}$ by taking the logarithm of both sides.

$\log y^3 = \log\left(\frac{z}{7}\right)$ or $\ln y^3 = \ln\left(\frac{z}{7}\right)$

$3\log y = \log z - \log 7$ or $3\ln y = \ln z - \ln 7$

2. Rewrite the formula $y = 59x^{3.2}$ by taking the logarithm of both sides.

$\log y = \log(59x^{3.2})$

$\log y = \log 59 + \log x^{3.2}$

$\log y = \log 59 + 3.2\log x$

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3. If $\log(a) = 2.3$ and $\log(b) = 4.2$, what is $\log\left(\frac{\sqrt[3]{b}}{a}\right)$?

$$\log\left(\frac{\sqrt[3]{b}}{a}\right) = \log(\sqrt[3]{b}) - \log(a)$$
$$= \frac{1}{3}\log b - \log a$$
$$= \frac{1}{3}(4.2) - 2.3$$
$$= 1.4 - 2.3 = \boxed{-.9}$$

4. Solve $2^x - 1 = 4$

$$2^x = 5$$
$$\ln 2^x = \ln 5$$
$$x \ln 2 = \ln 5$$
$$\boxed{x = \frac{\ln 5}{\ln 2}} \quad \text{or} \quad \boxed{x = 2.3219}$$

5. Solve $\ln(x-6) + 11 = 13$.

$$\ln(x-6) = 2$$
$$e^2 = x - 6$$
$$\boxed{e^2 + 6 = x} \quad \text{or} \quad \boxed{x = 13.389}$$

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6. What is the future value, in 5 years, of an initial investment of $12,000 at an annual interest rate of 6%, compounded monthly?

$$FV = 12{,}000\left(1 + \frac{.06}{12}\right)^{12(5)}$$

$$= 12{,}000(1 + .005)^{12(5)}$$

$$= 12{,}000(1.005)^{60}$$

$$= \boxed{\$16{,}186.20}$$

7. The value V of an initial investment of $3000 compounded continuously at a 4% rate of return is given by the function $V(t) = 3000e^{.04t}$, where t is the time in years from the initial investment. Approximately how many years would it take for this investment to grow to a value of $30,000?

$$30{,}000 = 3000e^{.04t}$$

$$10 = e^{.04t}$$

$$\ln 10 = \ln e^{.04t}$$

$$\ln 10 = .04t$$

$$\boxed{t = 57.56 \text{ years}}$$

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8. The revenue function for the sale of a certain product is given by $R(x) = -x^3 + 86x^2 - 700x$, where x is the number of units sold. If the sale of 70 units leads to \$29,400 in revenue, what is another number of units that leads to the same amount of revenue?

$$\begin{array}{r|cccc} 70 & -1 & 86 & -700 & -29,400 \\ & \downarrow & -70 & 1120 & 29,400 \\ \hline & -1 & 16 & 420 & 0 \end{array}$$

$-x^2 + 16 + 420 = 0.$

$(-x - 14)(x - 30) = 0.$

$x = -14$ or $\boxed{x = 30}$

$\boxed{30 \text{ units}}$

9. Find 2 possible 4th degree polynomials with single roots at $x = -1$ and $x = 5$, and a double root at $x = 4$. Write the polynomials in standard form $a_n x^n + \ldots + a_1 x + a_0$.

Answer 1: $x^4 - 12x^3 + 43x^2 - 24x - 80$

$(x+1)(x-5)(x-4)^2$

$= (x^2 - 4x - 5)(x^2 - 8x + 16)$

$= x^4 - 8x^3 + 16x^2$
$\quad - 4x^3 + 32x^2 - 64x$
$\quad\quad - 5x^2 + 40x - 80$

$x^4 - 12x^3 + 43x^2 - 24x - 80$

Answer 2: _____

any nonzero constant multiplied through answer 1:

i.e:

$-x^4 + 12x^3 - 43x^2 + 24x + 80$

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10. Given that $x = 4$ and $x = -4$ are roots of the polynomial $P(x) = 2x^4 + 11x^3 - 53x^2 - 176x + 336$, find all other roots, both real and complex, of $P(x)$.

4 | 2 | 11 | −53 | −176 | 336
↓ | | 8 | 76 | 92 | −336
| 2 | 19 | 23 | −84 | 0

−4 | 2 | 19 | 23 | −84 |
↓ | | −8 | −44 | 84 |
| 2 | 11 | −21 | 0

$2x^2 + 11x - 21 = 0$.

$(2x - 3)(x + 7) = 0$

$x = 3/2$ or $x = -7$

11. Let $R(x) = x^3 + 2x^2 - 13x + 10$. Is $x - 2$ a factor of $R(x)$? How do you know?

2 | 1 | 2 | −13 | 10
↓ | | 2 | 8 | −10
| 1 | 4 | −5 | 0.

Yes, since remainder is zero when dividing R(x) by x−2.

You could use long division also.

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12. The profit function for a certain product is given by $P(x) = \cancel{1660}1600x - x^3$, where x is the number of units sold. What values of x result in zero profit?

$1600x - x^3 = 0$

$x(1600 - x^2) = 0$

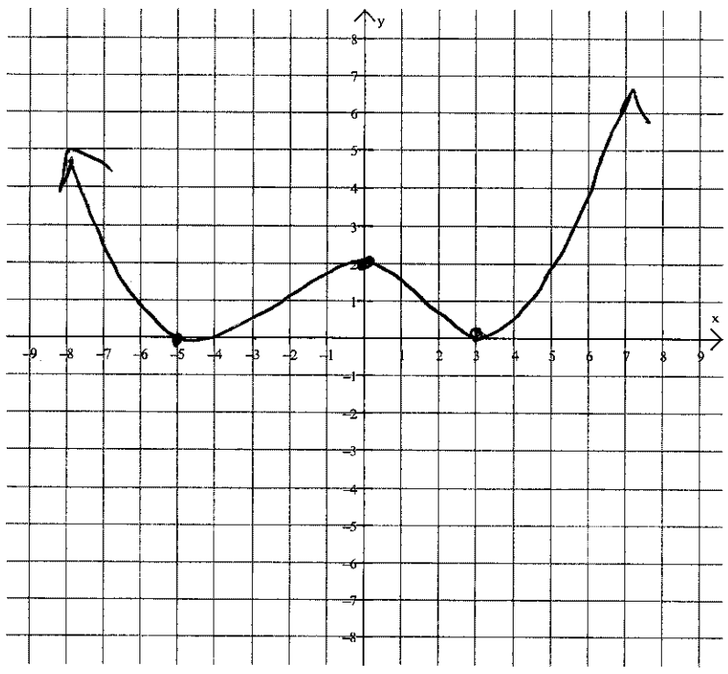
$x(40-x)(40+x) = 0$

$x = 0, \quad x = 40, \quad x = -40.$

0 or 40 units

13. Sketch a polynomial on the grid below with the given characteristics:

a) Even degree
b) Positive leading coefficient
c) Positive constant term
d) A repeated root at $x = 3$
e) A repeated root at $x = -5$



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14. What is the horizontal asymptote of the function $f(x) = 5^x + 10$? Explain how you arrived at your answer.

$y=0$ would be the horizontal asymptote of $y=5^x$.

"$+10$" means $y=5^x$ is vertically shifted by 10 units.

So, $y=10$ is the answer

15. What is the domain of the function $D(x) = 25\log(x-16)$?

$x-16 > 0$.

$x > 16$

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16. The volume of a box can be described by the formula $V(x) = 256x - 64x^2 + 4x^3$, where x is the height of the box in inches.

a) Find all the roots of *V(x)*. Are any of the roots repeated roots?

$(256 - 64x + 4x^2)x = 0$.

$(64 - 16x + x^2)4x = 0$.

$(x-8)(x-8)4x = 0$

$x = 8$ (yes, it repeats)

$x = 0$

b) What is *V(4)?*

$V(4) = 256(4) - 64(4)^2 + 4(4)^3$

$= 256$ inches^3

or.

4 | 4 | −64 | 256 | 0

↓ | | 16 | −192 | 256.

4 | −48 | 64 | 256.

$V(4) = 256$.

(remainder theorem).