

Name: KEY

Recitation Instructor and Time: _____

Studio College Algebra – Exam 2
October 14, 2008

Directions: There are 16 problems on this exam. Please show all your work.

1. Solve: $2x^2 - 9x - 5 = 0$.

Factoring

$$(2x + 1)(x - 5) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad x - 5 = 0$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2} \quad \text{or} \quad x = 5}$$

With quadratic formula:

$$a = 2$$

$$b = -9$$

$$c = -5$$

$$x = \frac{9 \pm \sqrt{81 - 4(2)(-5)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{121}}{4}$$

$$= \frac{9 \pm 11}{4} = \frac{20}{4} \quad \text{or} \quad \frac{-2}{4}$$

$$\boxed{x = 5 \quad \text{or} \quad -\frac{1}{2}}$$

2. Write $x^2 + 6x - 10$ in the form $a(x - h)^2 + k$.

Method 1

$a = 1$ in this example. So, $(x - h)^2 + k = x^2 - 2hx + h^2 + k$.

So, $-2h = 6$ and $h = -3$, while $h^2 + k = -10$, so

$$(-3)^2 + k = -10$$

$$9 + k = -10$$

$$k = -19.$$

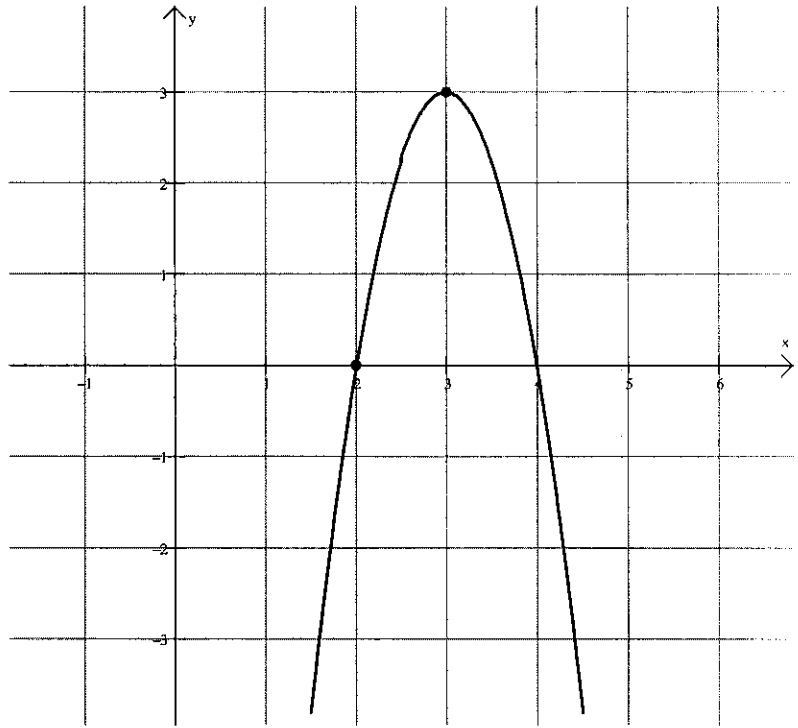
Answer: $\boxed{(x + 3)^2 - 19}$

Method 2: $x^2 + 6x - 10 = \underbrace{x^2 + 6x + 9}_{(x+3)^2} - 9 - 10$
 $= \boxed{(x + 3)^2 - 19}$

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3. The parabola given below has vertex at (3,3) and passes through the point (2,0). Find an equation of the parabola in the form $ax^2 + bx + c$.

$$\begin{aligned}y &= a(x-h)^2 + k \\0 &= a(2-3)^2 + 3 \\0 &= a(1) + 3 \\-3 &= a \\y &= -3(x-3)^2 + 3 \\y &= -3(x^2 - 6x + 9) + 3 \\y &= -3x^2 + 18x - 27 + 3 \\y &= -3x^2 + 18x - 24\end{aligned}$$



4. If $P(x) = .6x^2 + 120x + 100$, for what value of x does $P(x)$ attain a minimum value? What is the minimum value?

$$\text{Vertex: } (h, k) \text{ where } h = \frac{-b}{2a} \text{ and } k = P(h)$$

$$\text{So, } h = \frac{-120}{2(.6)} = \frac{-120}{1.2} = -100$$

$$P(-100) = .6(100)^2 + 120(100) + 100$$

$$P(-100) = -5900$$

$P(x)$ attains a minimum value of -5900 at $x = -100$.

5. Let $f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 1$. Answer the following:

a) Compute $f(x)g(x)$ (simplify completely).

$$(x^2 + 2x - 3)(3x - 1) = 3x^3 + 6x^2 - 9x - x^2 - 2x + 3 = \boxed{3x^3 + 5x^2 - 11x + 3}$$

or:

	x^2	$2x$	-3
$3x$	$3x^3$	$6x^2$	$-9x$
-1	$-x^2$	$-2x$	3

$$\boxed{3x^3 + 5x^2 - 11x + 3}$$

b) Compute $g(f(x))$ (simplify completely).

$$\begin{aligned}
 \cancel{g(3x-1)} &= \cancel{(3x-1)^2 + 2(3x-1) - 3} = \cancel{3(x^2 + 2x - 3) - 1} \\
 &= \cancel{9x^2 - 6x + 1 + 6x - 2 - 3} = \boxed{3x^2 + 6x - 10} \\
 &= \boxed{9x^2 - 4} \quad \text{if they have } (3x-2)(3x+2) \\
 &\quad \text{That's ok.}
 \end{aligned}$$

6. Given $h(x) = 2x^2 - 5x + 1$, find $h(h(2))$.

$$\begin{aligned}
 h(2) &= 2(2)^2 - 5(2) + 1 \\
 &= 8 - 10 + 1 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 h(h(2)) &= h(-1) = 2(-1)^2 - 5(-1) + 1 \\
 &= 2 + 5 + 1 \\
 &= \boxed{8}
 \end{aligned}$$

7. Given $f(x) = x^2 + 5$ on the domain $x \geq 0$, find $f^{-1}(x)$.

$$y = x^2 + 5$$

$$x = y^2 + 5$$

$$x - 5 = y^2$$

$$\sqrt{x-5} = y$$

$$f^{-1}(x) = \sqrt{x-5}$$

$$x \geq 5$$

8. Solve: $|2x+1| = 3x+5$

$$2x+1 = 3x+5$$

$$-4 = x$$

$$\text{or } 2x+1 = -3x-5$$

$$\text{or } 5x = -6$$

$$x = -\frac{6}{5}$$

Check: $|2(-4)+1| = 7$

$$3(-4)+5 = -7$$

$x = -4$ doesn't
work

$$|2(-\frac{6}{5})+1| = |-\frac{12}{5}+\frac{5}{5}| = \frac{7}{5}$$

$$3(-\frac{6}{5})+\frac{5}{5} = -\frac{18}{5}+\frac{5}{5} = -\frac{13}{5}$$

$$x = -\frac{6}{5} \text{ works}$$

9. Solve: $|3x-8| < 12$

$$-12 < 3x-8 < 12$$

$$-12 < 3x-8$$

$$-4 < 3x$$

$$-\frac{4}{3} < x$$

$$x > -\frac{4}{3}$$

$$\text{and } 3x-8 < 12$$

$$3x < 20$$

$$x < \frac{20}{3}$$

$$\text{and } x < \frac{20}{3}$$

$$-\frac{4}{3} < x < \frac{20}{3}$$

10. Solve and check your answers: $x+3 = \sqrt{3x+7}$

$$(x+3)^2 = (\sqrt{3x+7})^2$$

$$x^2 + 6x + 9 = 3x + 7$$

$$x^2 + 3x + 2 = 0$$

$$(x+3)(x+1) = 0$$

$$x = -2 \text{ or } x = -1$$

Check: $-2 + 3 = 1$

$$\sqrt{3(-2)+7} = \sqrt{1} = 1$$

$x = -2$ works

$$-1 + 3 = 2$$

$$\sqrt{3(-1)+7} = \sqrt{4} = 2$$

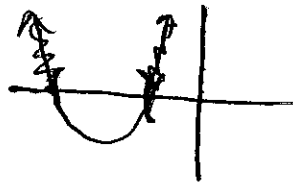
$x = -1$ works

11. Solve: $x^2 + 3x + 4 > 2$

$$x^2 + 3x + 2 > 0$$

$$(x+2)(x+1) > 0$$

Graphically:



$x^2 + 3x + 2$ is positive
when $x > -1$ or $x < -2$.

~~True~~ False ~~True~~

Test
 $x = -5$
 $(-5+2)(-5+1)$
 $= (-3)(-4)$
 $= 12 > 0$
True

-2 -1

Test
 $x = -1.5$
 $(-1.5+2)(-1.5+1)$
 $= (-.5)(-.5)$
 $= -.25$
 $-.25 \not> 0$
False

test $x = 0$.
 $(0+2)(0+1)$
 $= (2)(1)$
 $= 2 > 0$
True

Answer: $x > -1$ or
 $x < -2$

12. The cost of producing x units in a manufacturing plant is given by the function $C(x) = 0.30(x+3)^2 + 100$. The number of units produced in t hours is given by $x(t) = 5t$.

a) Find $C(x(t))$, and simplify completely.

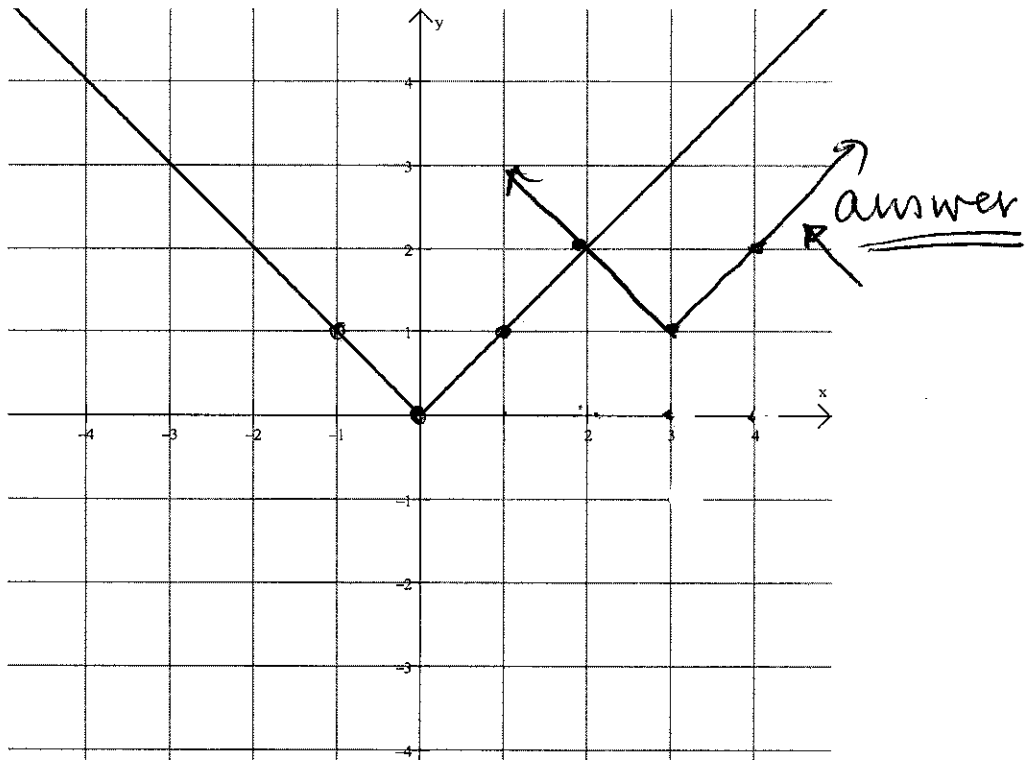
$$\begin{aligned} C(5t) &= .30(5t+3)^2 + 100 \\ &= .30(25t+30t+9) + 100 \\ &= 7.5t^2 + 9t + 2.7 + 100 = \boxed{7.5t^2 + 9t + 102.7} \end{aligned}$$

b) Find $C(x(4))$.

$$x(4) = 5(4) = 20$$

$$\begin{aligned} C(x(4)) &= C(20) = .3(20+3)^2 + 100 \\ &= \boxed{\$258.70} \end{aligned}$$

13. Given $f(x)$ below, graph $f(x-3)+1$ on the same set of axes.



14. Suppose the number of on-duty injuries associated with a certain occupation could be modeled by the function $I(x) = 124.8x^{-.346}$, where x represents the numbers of years after 1990.

a) Find $I(7)$ and label your answer.

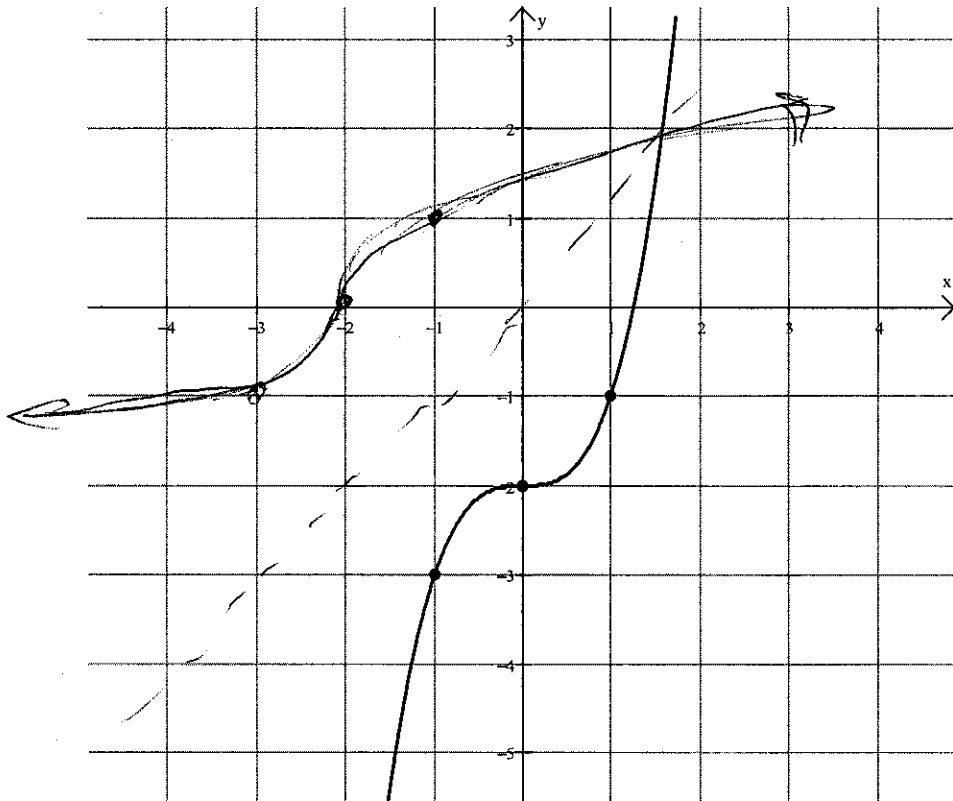
$$I(7) = 124.8(7^{-.346})$$

$$\approx 64 \text{ injuries}$$

b) Between the years 1995 and 2000, are the number of injuries increasing or decreasing? How do you know?

The power on x is negative, signifying decay; (decreasing) one could also observe a table of x, y values and see the same pattern.

15. Given $f(x)$ below, draw $f^{-1}(x)$ on the same plot.



16. The revenue function associated with a certain product is given by $R(x) = 105x$ dollars, and the cost function is given by $C(x) = 1000 + 3x + .1x^2$ dollars, where x is the number of units that are produced and sold.

- a) What is the profit function, $P(x)$ associated with this situation?

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 105x - (1000 + 3x + .1x^2) \\ &= 102x - .1x^2 - 1000 \end{aligned}$$

$$P(x) = -.1x^2 + 102x - 1000$$

- b) What is the profit on the production and sale of 100 units?

$$\begin{aligned} P(100) &= -.1(100)^2 + 102(100) - 1000 \\ &= \boxed{\$8200} \end{aligned}$$