Name: key

Recitation Instructor and Time: _____

Studio College Algebra – Exam 3
November 11, 2008

Please show all your work for full credit. Every problem is worth 5 points.

1. Rewrite the formula $y = 103x$ by taking the logarithm of both sides, and simplify your answer.

$\ln y = \ln(103x)$
$\ln y = \ln 103 + \ln x$

or

$\log y = \log(103x)$
$\log y = \log 103 + \log x$

2. Solve for S in the following equation: $\ln\left(\frac{S}{1500}\right) = 2$

$e^2 = \frac{S}{1500}$

$1500e^2 = S$

$S = 11,083.5842$

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3. If $\log(a) = 2.4$ and $\log(b) = 4.8$, what is $\log\left(\sqrt[3]{ab}\right)$?

$$\frac{1}{3}[\log ab] = \frac{1}{3}[\log a + \log b]$$
$$= \frac{1}{3}[2.4 + 4.8]$$
$$= \frac{1}{3}(7.2)$$
$$= \boxed{2.4}$$

4. Solve $3 + 2^x = 10$

$$2^x = 7$$
$$\ln 2^x = \ln 7$$
$$x \ln 2 = \ln 7$$
$$\boxed{x = \frac{\ln 7}{\ln 2}}$$

5. Solve $3\ln(x+5) = 18$.

$$\ln(x+5) = 6$$
$$e^6 = x + 5$$
$$\boxed{e^6 - 5 = x}$$

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6. What is the future value in 10 years of an initial investment of $500 at an annual interest rate of 6%, compounded monthly?

$$FV = PV\left(1 + \frac{r}{n}\right)^{nt}$$

$$FV = 500\left(1 + \frac{.06}{12}\right)^{12 \cdot 10}$$

$$FV = \$909.70$$

7. A certain type of bacteria grows according to the function $P(x) = 3500(2^x)$, where x is the number of hours that have passed by. How many bacteria will there be after 2 hours?

$$P(2) = 3500(2^2)$$

$$= 3500(4)$$

$$= 14{,}000 \text{ bacteria}$$

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8. The number of widgets (in thousands) demanded each year is given by the formula $D(x) = 5 + 10\log(x+3)$, where x represents the number of years after 1980, and $x > 0$. In what year were 15,000 widgets demanded?

$15 = 5 + 10\log(x+3)$

$10 = 10\log(x+3)$

$1 = \log(x+3)$

$10 = x+3$

$x = 7$

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9. Find 2 possible 3rd degree polynomials with single roots at $x = 5, x = -1$, and $x = 2$. Write the polynomials in standard form $a_n x^n + \ldots + a_1 x + a_0$ (in other words, multiply everything out). (Show your work, but put answers in the lines given below).

First answer: $x^3 - 6x^2 + 3x + 10$

Second answer: any nonzero multiple of the 1st answer

$(x-5)(x+1)(x-2)$

$= (x^2 - 4x - 5)(x-2)$

$= x^3 - 4x^2 - 5x - 2x^2 + 8x + 10$

$= x^3 - 6x^2 + 3x + 10$

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10. Given that -5 is a solution, find all solutions, both real and complex, of the following equation: $x^3 + 5x^2 + 3x + 15 = 0$.

$$\begin{array}{r|cccc} -5 & 1 & 5 & 3 & 15 \\ & \downarrow & -5 & 0 & -15 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2 + 3 = 0$

$x^2 = -3$

$x = \pm\sqrt{3}$ $\quad$ $x = \pm\sqrt{3}i$ & $x = -5$

11. Is $x - 3$ a factor of $x^3 - 5x^2 + 10x - 12$? How do you know?

$$\begin{array}{r|cccc} 3 & 1 & -5 & 10 & -12 \\ & \downarrow & 3 & -6 & 12 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

Yes, it's a factor; the remainder = 0.

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12. Given that $x=2$ and $x=-1$ are roots of the following polynomial, find all other roots, real and complex, of the polynomial: $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

$$\begin{array}{r|cccc} 2) & 1 & -2 & -13 & 14 & 24 \\ & & \downarrow 2 & 0 & -26 & -24 \\ \hline -1) & 1 & 0 & -13 & -12 & \\ & & \downarrow -1 & 1 & 12 & \\ \hline & 1 & -1 & -12 & 0 & \end{array}$$

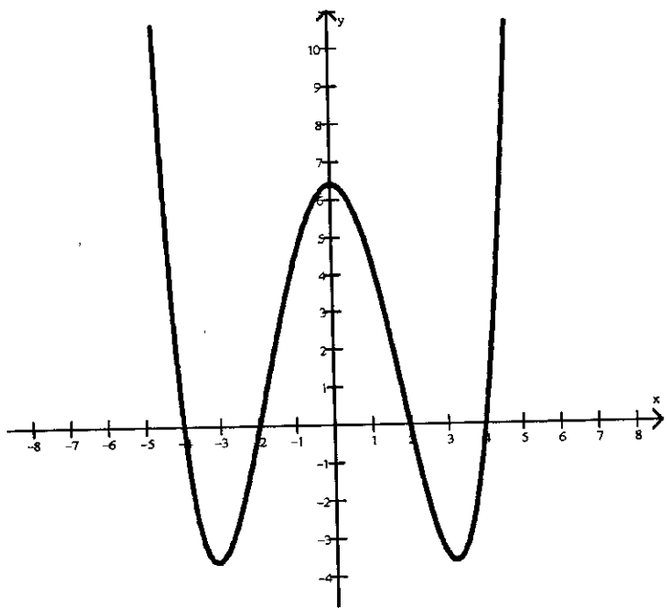
$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, x = -3$

$x = 2, x = -1$

13. Given the graph on the right, decide whether the following statements are **True or False**. You may assume nothing interesting happens outside the window shown.



a) This polynomial has a positive leading coefficient.

TRUE

b) The polynomial has a positive constant term.

TRUE

c) The polynomial does not have any repeated roots.

TRUE

d) The polynomial has odd degree.

FALSE

e) As x tends to both positive and negative infinity, the polynomial tends towards negative infinity.

FALSE

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14. What is the domain of the function $f(x) = 4 + 3\ln(-5x + 2)$?

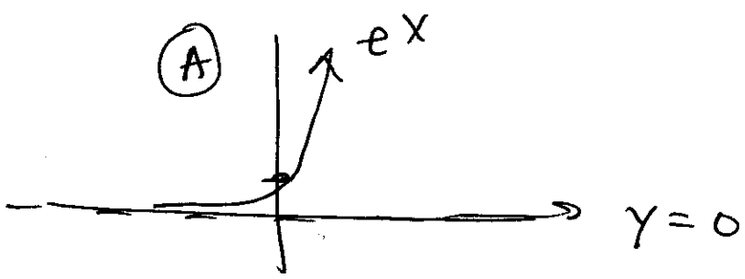
$-5x + 2 > 0$

$-5x > -2$

$\boxed{x < 2/5}$

15. What is the horizontal asymptote of the function $f(x) = e^x + 5$?

$\boxed{y = 5}$



$f(x) = e^x + 5$

→ shift the graph of (A) up by 5 units.

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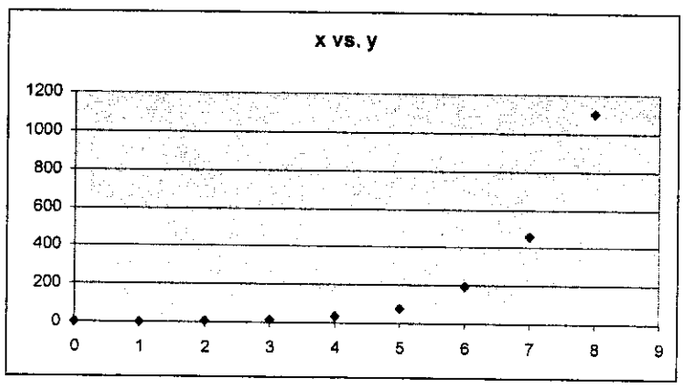
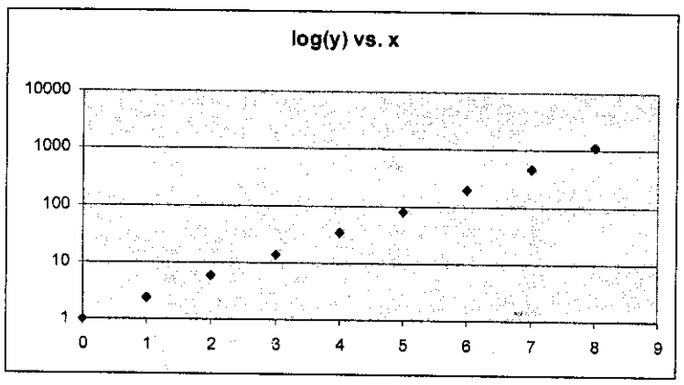
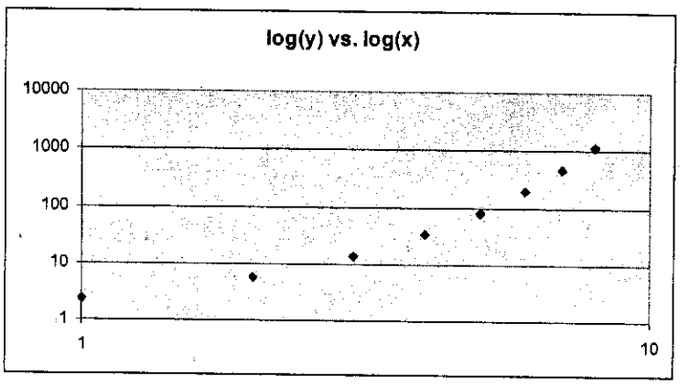
16. Some students have a data set, for which they create standard, log-log, and semi-log plots. (The plots are given below). Would a power or exponential model would be an appropriate fit for the data set? How do you know?







Exp. is better.

The logy vs. x (semilog plot) presents data in a linear pattern.