Sections 5.3/5.4/5.5 - Properties of Logarithms

Note Title 10/21/2009

→ So far, we've solved equations like

$3x \cdot 7 = 8$ (linear)

$3x^2 + 6x - 9 = 0$ (quadratic)

Today: $e^{3x+1} = 7$

Definition of Logarithm:

We say that $\log_{10}(m) = b$ if and only if $10^b = m$.

We say that $\ln(m) = b$ if and only if $e^b = m$

Ex] Write $e^{3a} = b$ in logarithmic form.

Answer $\ln(b) = 3a$

Properties of Logarithms

(assume that a, b are positive)

① $\log(ab) = \log(a) + \log(b)$

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② $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$

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③ $\log(a^n) = n\log(a)$

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Ex) Expand $\ln\left(\frac{ab^3}{c}\right)$

Rule 2: $\ln\left(\frac{ab^3}{c}\right) = \ln(ab^3) - \ln(c)$

Rule 1: $= \ln(a) + \ln(b^3) - \ln(c)$

Rule 3: $= \boxed{\ln(a) + 3\ln(b) - \ln(c)}$

Ex) Expand $\ln\left((2.5)(e^{.034x})\right)$

* This sort of thing appears in Studio **

Rule 1 $= \ln(2.5) + \ln(e^{.034x})$

$= \ln(2.5) + .034x(\ln e)$

$= \boxed{\ln(2.5) + .034x}$

$\ln e = \boxed{1}$

Ex) (Online HW)

Using $\ln 2 = .693$ $\quad \ln(a) =$ [redacted]

$\ln 3 = 1.099$ $\quad \ln(b) = 2.298$

Compute $\ln\left(\frac{3}{a^4}\right)$.

$$\ln\left(\frac{3}{a^4}\right) = \ln(3) - \ln(a^4)$$

$$= \text{[redacted]} - 4\,\text{[redacted]}$$

$$= 1.099 - 4(1.236)$$

$$= \boxed{-3.845}$$

Ex) Compute $\ln(a\sqrt{b})$

$$= \ln(a) + \ln(\sqrt{b})$$

$$= \ln(a) + \ln b^{1/2}$$

$$= \ln(a) + \frac{1}{2}\ln b$$

and so on....

What is $\sqrt[3]{a}$? $\quad a^{1/3}$

$\sqrt[5]{b}$? $\quad b^{1/5}$

Ex] Solve for x: $e^{3x} = 7$

* "TAKE THE NATURAL LOGARITHM OF BOTH SIDES" *

$$\ln(e^{3x}) = \ln(7)$$

$$3x \ln(e) = \ln(7)$$

$$3x = \ln(7) \longrightarrow \text{linear equation}$$

$$\boxed{x = \frac{\ln(7)}{3}}$$ Exact answer

Ex Solve for x:

$$4^x + 9 = 12$$

$$4^x = 3$$

or

$$\log 4^x = \log 3$$

$$x \log 4 = \log 3$$

$$x = \frac{\log 3}{\log 4}$$

$$\ln 4^x = \ln 3$$

$$x \ln 4 = \ln 3$$

$$x = \frac{\ln 3}{\ln 4}$$

Same answer!

Ex) Solve for x:

$$3\ln(x) - 3 = -2$$

$$3\underbrace{\ln(x)} = 1$$

$$\ln_e(x) = \frac{1}{3}$$

Convert to exponential form:

$$\boxed{e^{1/3} = x}$$

exact answer!

Section 5.5. Exponential Functions & $

Present Value: value today

Future Value: The amount to which an investment grows over a period of time.

Situation: You have $1000 today. How much will you have in 5 years if interest is compounded annually at an interest rate of 4%?

Amount after 1 year: $\underline{1000} + .04(1000)$

$= 1000[1 + .04]$

\$1040 $= \underline{\underline{1000[1.04]}}$

Amount after 2 years: $1040 + .04(1040)$

$= \underbrace{1040}\underbrace{[1 + .04]}$

$= 1000[1.04][1.04]$

$= 1000[1.04]^2$

$\vdots$

Amount after 5 years $= 1000[1.04]^5$

$= \boxed{\$1216.65}$

FORMULAS: ① After $\underline{n}$ periods of compounding, where r is the interest rate at each compounding,

$$FV = PV(1+r)^n$$

future value

Present Value

Book: $$FV = PV\left(1+\frac{r}{k}\right)^{kt}$$

r: interest rate (annual)

k: # of compoundings / year

t: # of years.