# Section 6.4 / Roots of Polynomials

Note Title 11/4/2009

Factor $x^2 - 5x + 6$.

$$\underbrace{(x-2)}\underbrace{(x-3)}$$

$x-2$ is a linear factor of $x^2-5x+6$

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Consider $x^3 + x^2 - 9x - 9 = p(x)$

Find the zeros of $p(x)$.

$$x^3 + x^2 - 9x - 9 = 0 \qquad \left(\text{Solve for } x\right)$$

$$x^2(x+1) - 9(x+1) = 0$$

$$(x+1)(x^2-9) = 0$$

$$(x+1)(x-3)(x+3) = 0$$

Zeros are $x = -1$, $x = 3$, $x = -3$

That $x = 3$ is a zero of $p(x)$ means that $x-3$ is a linear factor of $p(x)$.

Ex) Given $f(x) = x^3 - 3x^2 + 2$, is $x+4$ a factor of $f(x)$?

NOT A Factor.

$$f(-4) = (-4)^3 - 3(-4)^2 + 2$$

$$= -64 - 3(16) + 2 = -64 - 48 + 2$$

Examples: Find all zeros (roots), both real & complex, of

$$p(x) = x^3 + 2x^2 + 9x + 18,$$

given that $x = -2$ is one zero.

$\boxed{\text{Technique 1}}$ Polynomial Long Division

$$\begin{array}{r} x^2 \qquad\qquad + 9 \\ x+2 \overline{\big) x^3 + 2x^2 + 9x + 18} \\ \underline{-(x^3 + 2x^2)} \qquad\qquad \\ 9x + 18 \\ \underline{-(9x + 18)} \\ \boxed{0} \end{array}$$

$x^2(x+2)$

$9(x+2)$

$$p(x) = x^3 + 2x^2 + 9x + 18 = (x+2)\underbrace{(x^2+9)}$$

$x^2 + 9 = 0$

$x^2 = -9$

$\boxed{x = \pm 3i}$ the other zeros.

(*) When the divisor is linear, one can use synthetic division — this is a bookkeeping device to keep track of the steps in long division.

**Technique 2**

coefficients in $p(x)$ (descending powers of $x$; if a power is skipped, put 0 in its place)

| | | | | |
|---|---|---|---|---|
| $-2$ | 1 | 2 | 9 | 18 |
| | $\downarrow$ | $-2$ | 0 | $-18$ |
| | 1 | 0 | 9 | $\boxed{0}$ |

Zero of the divisor

divisor was $x+2$

$\rightarrow x^2+9$

$$x^2+9=0 \qquad x=\pm 3i$$

# Iclicker #2

$x = 2$ is a zero $p(x) = x^3 - 8x^2 + 22x - 20$

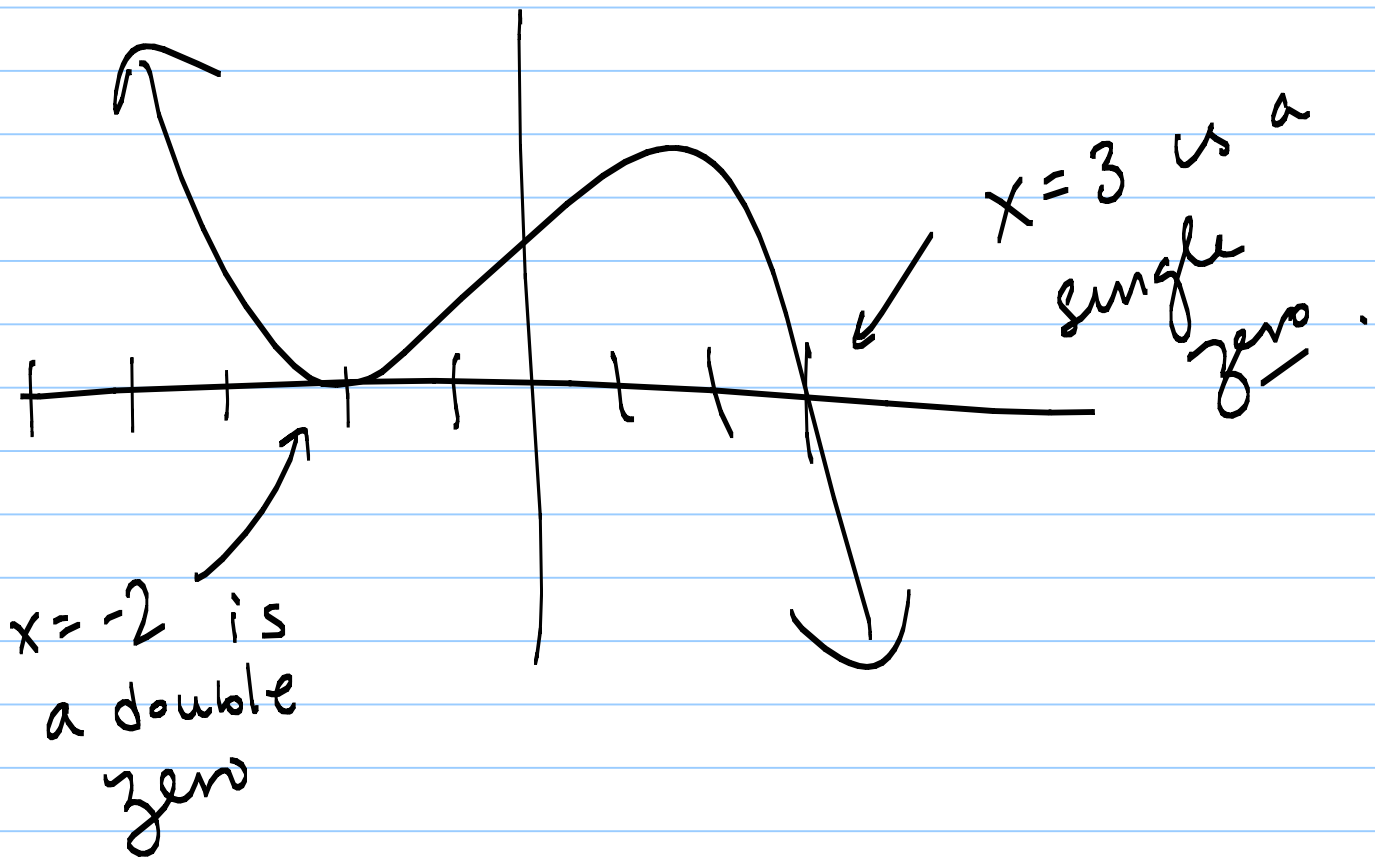
$$\begin{array}{r|rrrr} 2 & 1 & -8 & 22 & -20 \\ & \downarrow & 2 & -12 & 20 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

$$x^3 - 8x^2 + 22x - 20 = (x-2)\underbrace{(x^2 - 6x + 10)}_{\text{find the zeros of this.}}$$

$$\frac{6 \pm \sqrt{36 - 4(1)(10)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{-4}}{2} = \frac{6 \pm 2i}{2} = 3 + i \text{ or } 3 - i$$

Recall from last time the idea of multiple roots




$$p(x) = -(x+2)^2(x-3)$$

Ex) What is a polynomial with single roots at $x=-2$, $x=4$, and a double root at $x=0$?

Write answer in the form

$$a_n x^n + \cdots + a_1 x + a_0 .$$

$$(x+2)(x-4)x^2 \quad (\text{Multiply out})$$

$$(x^2-2x-8)x^2 = \boxed{x^4-2x^3-8x^2}$$

One possible answer

"Fundamental Theorem of Algebra"

If $f(x)$ is a polynomial function of degree $n \geq 1$, then $f$ has at least one complex zero.

$\longrightarrow$ Every polynomial function $f(x)$ of degree $n \geq 1$ has exactly <u>$n$ complex zeros.</u> (including multiplicities)

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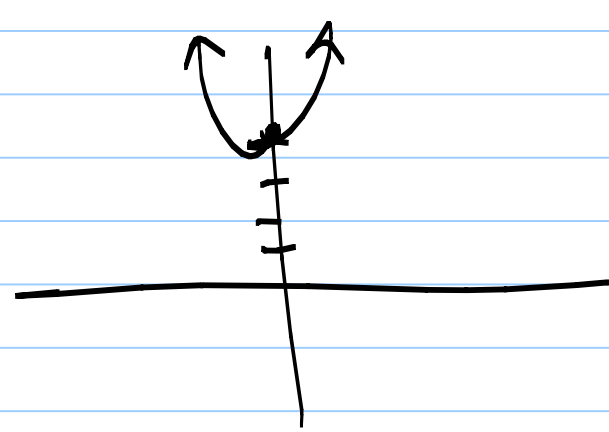
Consider $p(x) = x^2 + 4$.

2 complex zeros.



$x^2 + 4 = 0$

$x^2 = -4$

$\boxed{x = \pm 2i}$