Name: Key
Recitation Instructor:

College Algebra – Exam 3

July 16, 2009

Page 1 20 pts.	Page 2 28 pts.	Page 3 28 pts.	Page 4 24 pts.	TOTAL 100 pts

Directions: You will find 12 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. You have one hour to complete this exam.

1. (6 points) Given $p(x) = 5x^4 - 3x^3 + 7x^2 - 2x - 9$, use the Rational Zeros theorem and list all possible rational zeros of $p(x)$. (You do not have to find the actual zeros.)

$$\frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 5} \quad \frac{(\text{factors of constant})}{(\text{factors of leading coefficient})} \quad \left\{ \pm 1, \pm 3, \pm 9, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{9}{5} \right\}$$

2. (8 points) Find a polynomial with a single zero at $x = -4$ and double zero at $x = 3$. Write answer in the form $a_n x^n + \cdots + a_1 x + a_0$ (in other words, multiply out completely).

$$(x+4)(x-3)(x-3)$$

$$= (x+4)(x^2-6x+9) = \begin{matrix} x^3 - 6x^2 + 9x \\ +4x^2 - 24x + 36 \end{matrix} = \boxed{x^3 - 2x^2 - 15x + 36}$$

3. (6 points) Rewrite $y = \ln(x+2)$ in exponential form.

$$\boxed{e^y = x+2}$$

4. (16 points) Consider the rational function $r(x) = \dfrac{(4x+5)(4x-5)}{x^2-13x+12}$.

(a) Find the vertical asymptotes of $r(x)$.

$x^2-13x+12 = (x-12)(x-1) = 0.$ $\boxed{\begin{matrix} x=12 \\ x=1 \end{matrix}}$

(b) Find the horizontal asymptote of $r(x)$.

degree same $\rightarrow$ Lead coefficient numerator: 16.
Lead coefficient denominator: 1

$\boxed{y=16}$

(c) Find the zeros of $r(x)$.

$4x+5=0$ $\quad$ $4x-5=0$

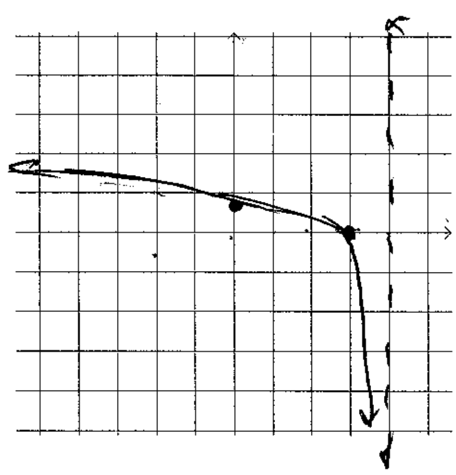
$\boxed{x=-5/4}$ $\quad$ $\boxed{x=5/4}$

(d) Find the y-intercept of $r(x)$.

$\boxed{r(0) = \frac{-25}{12}}$

5. (6 points)

(a) Graph the function $p(x) = \log(4-x)$ on the following window: $-5 \leq x \leq 5$, $-5 \leq y \leq 5$. Label all intercepts and identify any asymptotes.



x-intercept: $(3, 0)$
y-intercept:
$(0, \log 4)$
$\approx (0, .6021)$

(b) (6 points) What is the domain of $p(x) = \log(4-x)$?

$4-x > 0.$

$4 > x$

$\boxed{x<4}$

6. (6 points) Find the inverse of the function $f(x) = 5 - x^3$.

$$y = 5 - x^3$$
$$x = 5 - y^3$$
$$y^3 = 5 - x$$
$$y = \sqrt[3]{5-x}$$

$$\boxed{f^{-1}(x) = \sqrt[3]{5-x}}$$

7. (6 points) Expand using properties of logarithms: $\log(ab\sqrt{c})$. (You may assume that a, b and c are positive.)

$$\log a + \log b + \log \sqrt{c}.$$
$$= \log a + \log b + \log c^{1/2}$$
$$= \boxed{\log a + \log b + \frac{1}{2} \log c}$$

8. (8 points) Solve the quadratic inequality $x^2 - 13x + 12 > 0$.

$$\boxed{x > 12 \text{ or } x < 1}$$

$$(x-12)(x-1) > 0$$

+   1   −   12   +

test $x = 0$.	test $x = 2$	test $x = 13$
$(0-12)(0-1)$	$(2-12)(2-1)$	$(13-12)(13-1)$
$= (-12)(-1)$	$= (-10)(1)$	$(1)(12)$
$= 12$	$= -10$	$= 12$

9. (8 points)

(a) Given $f(x) = e^x - 3$, find $f(0)$ and $f(\ln 3)$.

$$f(0) = e^0 - 3 = 1 - 3 = -2$$
$$f(\ln 3) = e^{\ln 3} - 3 = 3 - 3 = 0$$

(b) Does the graph of $f(x)$ have a horizontal asymptote? If so, what is it?

Yes: $\boxed{y = -3}$

10. (6 points) The number of gizmos (in thousands) demanded each year is given by the formula $D(x) = 4 + \log(x+5)$, where x represents the number of years after 1990, and $x > 0$. How many gizmos were demanded in the year 1998?

Plug in $x = 8$

$D(8) = 4 + \log(8+5) = 5.114$

Roughly $\boxed{5114 \text{ gizmos}}$

11. (18 points) Consider the polynomial $p(x) = x^3 + 3x^2 - 9x - 27$.

(a) Using the leading-term test, describe the end behavior of $p(x)$.

Odd degree positive leading coefficient: as $x \to \infty$ $p(x) \to \infty$ as $x \to -\infty$ $p(x) \to -\infty$.

(b) Given that $x = -3$ is a zero of $p(x)$, find the other zeros of $p(x)$.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & -9 & -27 \\ & \downarrow & -3 & 0 & 27 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3$ (double)

$x = 3$ (single)

(c) What is the y-intercept of $p(x)$?

$(0, -27)$

(d) Using the **Remainder Theorem**, find $p(1)$ and $p(-1)$.

$$\begin{array}{r|rrrr} 1 & 1 & 3 & -9 & -27 \\ & \downarrow & 1 & 4 & -5 \\ \hline & 1 & 4 & -5 & \boxed{-32} \end{array}$$

$p(1)$

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -9 & -27 \\ & \downarrow & -1 & -2 & 11 \\ \hline & 1 & 2 & -11 & \boxed{-16} \end{array}$$

$p(-1)$

(e) Using the information from parts (a)-(d), sketch $p(x)$ on the plot below.