Math100: Sample Final Exam A - Spring 07

1. Perform the division $(x^4 - 1) \div (x - 2)$.

2. For $f(x) = 2x^4 - 7x^3 + 11x^2 - 28x + 12$ find (a) the rational zeros, (b) the remaining zeros, and (c) factor f completely (use complex numbers if necessary).

3. Factor into linear factors (using complex numbers where necessary
a) $15x^2 - 19x + 6$ (b) $x^6 - 3x^4 - 10x^2$.

4. Give the slope intercept form of the equation of the straight line through the point $(1, -3)$ and perpendicular to the line $3x - 4y = 11$.

5. Solve the system of equations: $x^2 - 3y^2 = 1, \quad xy = 2$.

6. Solve the equation $\dfrac{5}{x-1} = \dfrac{3}{x-2} + \dfrac{1}{x^2 - 3x + 2}$.

7. $f(x) = \dfrac{x^3 - 4x}{x^2 - 1}$.

Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is f positive? (e) Where is f negative? Use this information to sketch the graph.

8. Find the domain, intercepts, asymptotes (all sorts), and test for symmetry

$$a)\ f(x) = \frac{x}{x^2 - 4}, \qquad b)\ g(x) = \frac{x^2 - 5x - 6}{3x^2 - x - 2},$$

9. Solve the inequality $|3x - 5| \leq 7$.

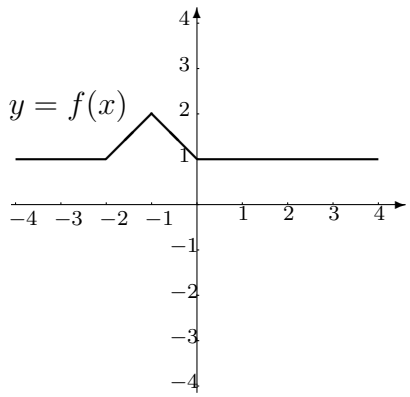
10. Find the (real) solutions of $\sqrt{2x + 3} + x = 6$.

11. Simplify (a) $\dfrac{\dfrac{3}{x-1} - \dfrac{2}{x+1}}{\dfrac{5}{x} - \dfrac{6}{x-1}}$, (b) $\dfrac{\dfrac{1}{x^2 + 2x} - \dfrac{1}{x^2 + 5x + 6}}{\dfrac{1}{x^2 - 9} - \dfrac{1}{x^2 + 3x}}$.

12. Simplify using only positive rational exponents (you may assume x, y, z are positive)
(a) $\dfrac{x^5 y^2 / z^2}{(xy^3)^2 (z/y)^3}$, (b) $y\sqrt[4]{y\sqrt[3]{y}}$.

13. Solve the inequality $\dfrac{2x}{x+3} \leq 1$.

14. Given the graph of $y = f(x)$ graph
(a) $f(x-2)$,
(b) $f(x)+2$,
(c) $f(2x)$,



15. (a) Write $\ln\left(\sqrt[3]{x^2}\, z^3/y^4\right)$ as a sum and difference of logs (assume x, y, z positive).
(b) Solve $5^{3x-2} = 7$.
(c) Solve $\log_3 x - \log_3(x-8) = 2$.

16. Use the method of elimination to solve the linear system: $3x + 2y = 25$, $5x - 4y = 5$.

17. (a) If $f(x) = x^2 + 1$ and $g(x) = x - 3$? What is $f \circ g$ and $g \circ f$?
(b) Find the inverse $f^{-1}(x)$ for the following functions or say why it does not exist.
(i) $f(x) = \frac{1}{2}(x^2 - 1)$, (ii) $f(x) = (x+1)^3 + 5$.

18. An artifact contains only 40% of its original carbon-14. If the half-life of carbon-14 is 5750 years, how old is the artifact? What percentage remains in something 10,000 year old?

19. Give the augmented matrix of the system of linear equations. Find (by calculator!) the reduced row echelon form and hence solve the system if possible.

$$\begin{aligned} 3x - 2y + z + 5w &= -3 \\ x + y - z + 3w &= -3 \\ 2x + 2y - 3z - w &= -2 \\ 5x + y - 5z + 2w &= -10. \end{aligned}$$

20. Write a matrix equation $AX = B$ for the system of linear equations. Find (by calculator) the inverse A^{-1} if it exists and hence if possible solve the system using matrix multiplication.

$$\begin{aligned} 2x - y + z &= 4 \\ 2x + z &= 1 \\ x - y + z &= 0. \end{aligned}$$

Math100: Sample Final Exam B - Spring 07

1. Perform the division $(3x^3 - 2x^2 + 3x + 2) \div (x^2 + x - 1)$.

2. For $f(x) = x^4 - 3x^3 + 5x^2 - x - 10$ find (a) the rational zeros, (b) the remaining zeros, and (c) factor f completely (use complex numbers if necessary).

3. Factor into linear factors (using complex numbers where necessary
a) $6x^2 - x - 15$ (b) $x^4 - 9$.

4. Give the slope intercept form of the equation of the straight line through the points $(1, -3)$ and $(3, -5)$.

5. Solve the system of equations: $x^2 - xy + y^2 = 3, \quad 2x + y = 5$.

6. Solve the equation $\dfrac{5}{x^2 - 4} = \dfrac{3}{x^2 - 3x + 2} + \dfrac{1}{x^2 + x - 2}$.

7. $f(x) = \dfrac{5x}{x^2 - 9}$.
Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is f positive? (e) Where is f negative? Use this information to sketch the graph.

8. Find the domain, intercepts, asymptotes (all sorts), and test for symmetry

$$a)\ f(x) = \frac{3x^2 - 9x}{x^2 - 4}, \qquad b)\ g(x) = \frac{x^2 - 5x - 6}{x + 2}.$$

9. Solve the inequality $|2x - 3| > 5$.

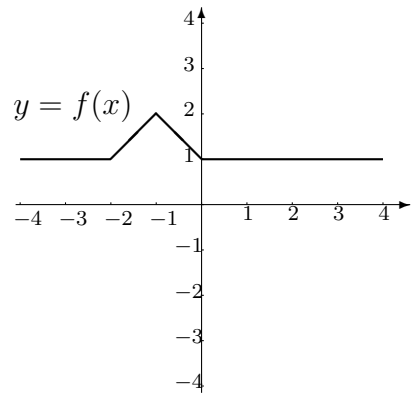
10. Find the (real) solutions of $\sqrt[4]{x^2 - x + 10} - 2 = 0$.

11. Simplify (a) $\dfrac{5}{x^2 - x - 6} - \dfrac{6}{x^2 - 9} + \dfrac{2}{x^2 + 5x + 6}$ (b) $\dfrac{\dfrac{3}{x+1} - \dfrac{2}{x}}{\dfrac{2}{x+2} - \dfrac{1}{x}}$.

12. Simplify using only positive rational exponents (you may assume x, y, z are positive)
(a) $\dfrac{\sqrt{x^3 y}}{\sqrt[5]{x^4 y^2}}$, (b) $\dfrac{(x^3 y/z)^2 y}{(xy^{-1})^3 z^{-3}}$.

13. Solve the inequality $\dfrac{x}{x+3} < \dfrac{x+1}{x+5}$.

14. Given the graph of $y = f(x)$ graph
(a) $f(x+2)$,
(b) $-2f(x)$,
(c) $\frac{1}{2}f(-x)$.



15. (a) Write $2\log_5(x+5) - \frac{1}{5}\log_5 x + 7\log_5 e$ as a single log.
(b) Solve $3e^{5-2x} = 7$.

(c) $\log_5 x + \log_5(x-4) = 1$.

16. Use the method of substitution to solve the linear system: $3x + 2y = 25$, $5x - 4y = 5$.

17. (a) What are the domains of $f(x) = 2/x$ and $g(x) = \sqrt{x-3}$? What is $f \circ g$ and $g \circ f$?

(b) Find the inverse $f^{-1}(x)$ for $f(x) = \dfrac{3}{x-5}$.

18. You deposit \$1000 in a bank account offering an annual interest rate of 4.33% compounded continuosly. How much will be in the account after 5 years? How many years before you have \$2000 in the account?

19. Give the augmented matrix of the system of linear equations. Find (by calculator!) the reduced row echelon form and hence solve the system if possible.

$$\begin{aligned} 3x - 2y + z &= 5 \\ x + y - z &= 7 \\ 9x - y - z &= 9. \end{aligned}$$

20. Write a matrix equation $AX = B$ for the system of linear equations. Find (by calculator) the inverse A^{-1} if it exists and hence if possible solve the system using a matrix multiplication.

$$\begin{aligned} x + 2y + w &= 6 \\ x + z &= -1 \\ x - y - z &= 1 \\ 2x + y + z + w &= 3. \end{aligned}$$