# Math 100 Final Exam

7:00pm–8:50pm, Wednesday, December 12, 2001

**Recitation Instructor:**____________________

**Recitation Day/Time:**____________________ **Name:**____________________________

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** Use back pages as scratch paper. Do problems at spaces provided.

| Total/100 | Page1/40 | Page2/40 | Page3/40 | Page4/40 | Page5/40 |
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**1.** (10pts) Simplify the fraction $\dfrac{x^3 - 5x^2 + 6x}{9 - x^2}$

**2.** (10pts) A bedroom is 3 feet and 6 inches longer than it is wide and has area 245 square feet. Find its dimensions.

**3.** (10pts) Solve the equation $\dfrac{4}{x(x+1)} + \dfrac{4}{x+1} = \dfrac{7}{x}$ and check your solution.

**4.** (10pts) Find all solutions (real and complex) of the equation $\dfrac{3}{x} = \dfrac{2}{x-2} + 1$.

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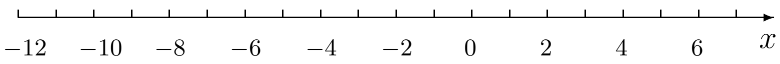
**5.** (10pts) Perform the operations and simply.

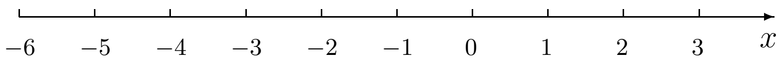
(a). $\dfrac{3}{x^2 - x - 2} - \dfrac{10}{x^2 + x - 6}$

(b). $\dfrac{\left[\dfrac{x^2 - 4}{x}\right]}{\left[\dfrac{(x-2)^2}{x}\right]}$

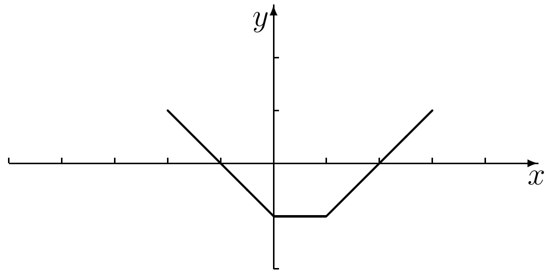
**6.** (10pts) Solve the inequality $|x + 2| > 4$ and graph the solution set:



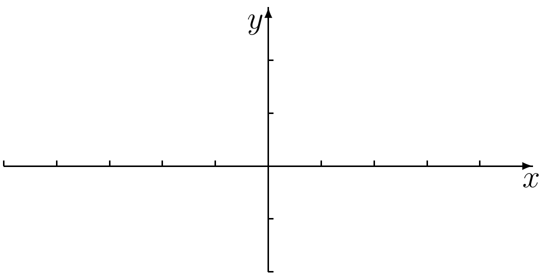

**7.** (10pts) Solve the inequality $x^3 + x^2 - 6x > 0$ and graph solution set.



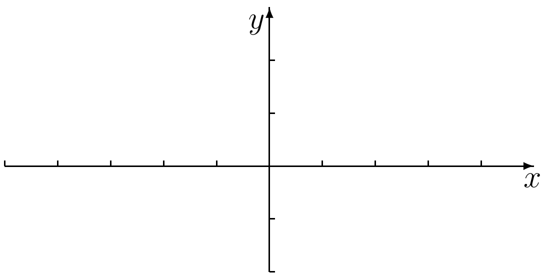

**8.** (10pts) The graph of a function $f(x)$ is on the right. Graph the given functions.




(a). $y = -f(x)$




(b). $y = 2f(x)$

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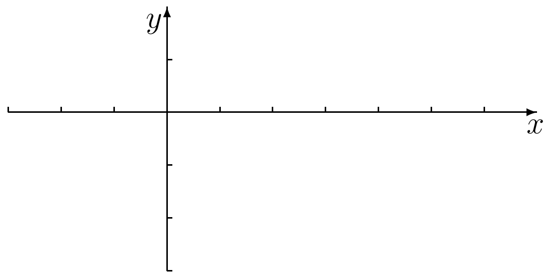
**9.** (10pts) On the day of a child's birth, a deposit of \$25,000 was made in a trust fund that pays \$8.75% interest, compounded continuously. Determine the balance in this account on the child's 25th birthday.

**10.**(10pts) Let $f(x) = x^3 - 7x^2 - 4x + 1$. Use your calculator to

(a). (2pts) Graph the function to see all turns. (copy the graph from your calculator carefully and write down your window settings.)




(b). Find all relative minimum(s) and relative maximum(s) (both $x$ and $y$-coordinates) (approximate to 3rd decimal place).

(c). List the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.
Intervals of increasing: Intervals of decreasing:

**11.**(10pts) For the polynomial $f(x) = x^4 - x^3 + x^2 - 3x - 6$, list all possible rational zeros using the rational zero test and find all rational zeros;

**12.**(10pts) For the function $f(x) = \dfrac{x^2 - 2x}{x+3}$, find its domain and all vertical, horizontal or slant asymptotes (write down their equations and clearly indicate each as vertical, horizontal, or slant). If there is a slant asymptote, write down its slope.

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**13.** (10pts) Given that $x^2+4$ is a factor, factor the polynomial $x^4-3x^3+6x^2-12x+8$ completely, using complex coefficients if necessary.

**14.** (10pts) For the quadratic function $f(x) = 2x^2+8x+7$
(a) determine the vertex of the parabola defined by the function $f(x)$;

(b) determine all $x$-intercepts (with exact values) of the graph of $f(x)$.

**15.** (10pts) Completely expand the logarithm as sum, difference, and/or constant multiples of simple logarithms ($x>1$).
$\ln\sqrt{x^3(x^2-1)} =$

**16.** (10pts) Solve the equation $e^{2x}-6e^x+5=0$ for $x$.

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**17.** (10pts) Use the substitution method to solve the system of equations $\begin{cases} x^2 + y &= 0 \\ x^3 - 6x - y &= 0 \end{cases}$

**18.** (10pts) Write the system of linear equations whose augmented matrix is given below (use $x, y, z$ as variables). Then use the back-substitution method to solve the system of linear equations.

$$\begin{bmatrix} 1 & -1 & 2 & \vdots & 4 \\ 0 & 1 & -1 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}.$$

**19.** (10pts) Compute the following, where $A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 2 & -3 \end{bmatrix}$.

(a) $AB =$

(b) $BA =$

**20.** (10pts) Given the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 1 & 1 \\ -4 & 4 & 3 \end{bmatrix}$, use a calculator to do the following (write the major operational key strokes you used): **Your calculator model is** ____________

(a). compute the reduced row-Echelon form of $A$;

(b). compute the inverse matrix $A^{-1}$ of $A$, (if $A^{-1}$ does not exist, say so).

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