Review Questions for Final Exam

Disclaimer: This is not a *practice exam.* It is meant to reflect the sort of material you will be expected to know on the exam, not to indicate the actual questions or length of the exam.

1. Solve the inequalities (a) $|x-5| \leq 7$, (b) $|2x-3| > 5$.

2. Simplify using only positive rational exponents (you may assume x, y, z are positive)

$$\frac{\sqrt{x^5y^2}/z^2}{(xy^3)^2(z/y)^3}.$$

3. Give the slope intercept form of the equation of the straight line through the point $(1, -3)$ and

(a) perpendicular to the line $3x - 4y = 11$.
(b) through the point $(3, -5)$.
(c) through the point $(1, -1)$.

4. Solve the equations

(a) $\dfrac{5}{x-1} = \dfrac{3}{x-2}$.

(b) $\dfrac{5}{x-1} = \dfrac{3}{x-2} + \dfrac{1}{(x-2)(x-1)}$.

(c) $\dfrac{5}{x-1} = \dfrac{3}{x-2} + 1$.

5. (a) Write $\ln\left(\sqrt[3]{x^2}\, z^3/y^4\right)$ as a sum and difference of logs (assume x, y, z positive).
(b) Write $2\log_5(x+5) - \frac{1}{3}\log_5 x + 7\log_5 e$ as a single log.
(c) Solve $\log_3 x + \log_3(x-8) = 2$.
(d) Solve $\log_3 x - \log_3(x-8) = 2$.
(e) Graph $y = \log_3(x-5)$. Label the intercepts and asymptotes.

6. Solve the inequalities (a) $x^4 < 4x^2$, (b) $\dfrac{x}{x+3} < \dfrac{x+1}{x-1}$.

7. Find all solutions (real and complex) of (a) $x^5 = 9x$, (b) $x^4 - 3x^2 - 10 = 0$.

8. You have \$1000 to deposit for 30 years. 'Premier bank' offers an annual interest rate of 4.35% compounded quarterly, while 'Credit One' offers 4.3% compounded continuosly. Which offer is best? How many years would it have to be deposited at Credit One to finish up with \$2000?

9. Find the domain, intercepts, asymptotes (all sorts), and test for symmetry

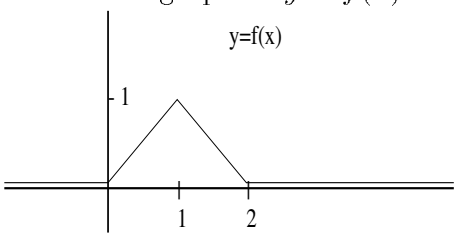
$$a)\ f(x) = \frac{x^2-3x}{x^2-4}, \qquad b)\ g(x) = \frac{x^2-5x-6}{x+2}, \qquad c)\ h(x) = \frac{x}{x^2-3}.$$

10. Find the (real) solutions of (a) $\sqrt{2x+3} + x = 6$, (b) $|2x+1| = 2 - x$,
(c) $\sqrt[4]{x^2 - x + 10} - 2 = 0$.

11. Find the rational zeros and hence factor completely (use complex numbers if necessary):

$$f(x) = 2x^4 - 7x^3 + 11x^2 - 28x + 12.$$

12. Given the graph of $y = f(x)$



sketch the graphs of a) $f(-x)$, b) $-f(x)$, c) $f(2x)$, d) $2f(x)$, e) $f(x+3)$, f) $f(x)+3$.

13. Give the partial fraction decomposition of $\dfrac{x^2 - 3x + 1}{x^3 - 3x^2 + 2x}$.

14. Give the augmented matrix of the system of linear equations. Find (by calculator!) the reduced row echelon form and hence solve the system.

$$(a) \quad \begin{aligned} 3x - 2y + z + 5w &= -3 \\ x + y - z + 3w &= -3 \\ 2x + 2y - 3z - w &= -2 \\ 5x + y - 5z + 2w &= -10. \end{aligned} \qquad (b) \quad \begin{aligned} 3x - 2y + z &= 5 \\ x + y - z &= 7 \\ 9x - y - z &= 9. \end{aligned}$$

15. Solve the equations (a) $\frac{1}{2}\, 3^{x-7} - 2 = 0$, (b) $e^{2x} - 5\, e^x - 6 = 0$.

16. (a) A company's profit (in millions of Hong Kong dollars) from producing x tons of its product is given by $P = 30x - 5 - 3x^2$. How much should it produce to maximise profits? (b) Write the parabola $y = 2x^2 - 8x + 5$ in standard form.

17. What is the domain of (a) $\dfrac{x^2}{x-7}$, (b) $\sqrt{x^3 - 5x^2 + 6x}$, (c) $\sqrt[3]{x-7}$.

18. Write $\dfrac{3x^3 - 2x^2 + 3x + 2}{x^2 + x - 1}$ as a polynomial and proper fraction.

19. Simplify $\dfrac{\dfrac{1}{x^2+2x} + \dfrac{1}{x^2+5x+6}}{\dfrac{1}{x^2-9} - \dfrac{1}{x^2+3x}}$.

20. Use elimination or substitution to solve the system of equations $2x^2 - y = 4$, $5x - 2y = 2$.

21. What are the domains of $f(x) = 3/x$ and $g(x) = \sqrt{x-3}$? What are $f \circ g$ and $g \circ f$?

22. Find the inverse $f^{-1}(x)$ for the following functions or say why it does not exist.
(a) $f(x) = \frac{1}{2}(x^2 - 1)$, (b) $f(x) = \dfrac{3}{x-5}$, (c) $f(x) = \sqrt[3]{2x+1}$.

Solutions

1. (a) $[-2, 12]$, (b) $(-\infty, -1) \cup (4, \infty)$.

2. $\dfrac{x^{\frac{1}{2}}}{y^2 z^5}$.

3. (a) $y = -\frac{4}{3}x - \frac{5}{3}$, (b) $y = -x - 2$, (c) $x = 1$.

4. (a) $x = 7/2$, (b) $x = 4$, (c) $x = \frac{5}{2} \pm \frac{\sqrt{11}}{2} i$.

5. (a) $\frac{2}{3} \ln x + 3 \ln z - 4 \ln y$.
(b) $\log_5 \left(\frac{e^7 (x+5)^2}{\sqrt[3]{x}} \right)$.
(c) $x = 9$. (d) $x = 9$.
(e) $y = \dfrac{\ln(x-5)}{\ln 3}$, $x = 5$, $(6, 0)$.

6. (a) $(-2, 0) \cup (0, 2)$, (b) $(-3, -\frac{3}{5}) \cup (1, \infty)$.

7. (a) $x = 0, \pm\sqrt{3}, \pm\sqrt{3}i$. (b) $x = \pm\sqrt{5}, \pm\sqrt{2}i$.

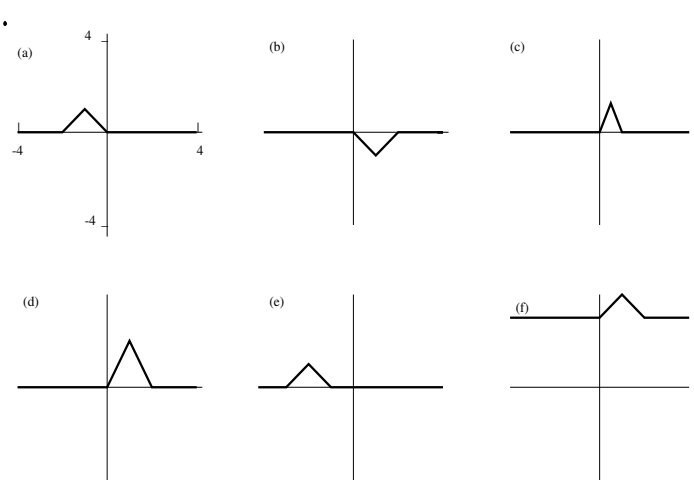
8. Premier bank \$3661.80 (Credit One \$ 3632.79). 16.1197 years.

9. (a) $x \neq \pm 2$, $(0, 0), (3, 0)$, $x = 2, x = -2$, $y = 1$, no symm.
(b) $x \neq -2$, $(0, -3), (-1, 0), (6, 0)$, $x = -2$, $y = x - 7$, no symm.
(c) $x \neq \pm\sqrt{3}$, (0,0), $x = \sqrt{3}, x = -\sqrt{3}$, $y = 0$, odd.

10. (a) $x = 3$. (b) $x = \frac{1}{3}, -3$. (c) $x = -2, 3$.

11. $x = 3, \frac{1}{2}$, $f(x) = (x-3)(2x-1)(x+2i)(x-2i)$.

12.



13. $\dfrac{\frac{1}{2}}{x} - \dfrac{\frac{1}{2}}{x-2} + \dfrac{1}{x-1}$.

14. (a) $\begin{pmatrix} 3 & -2 & 1 & 5 & -3 \\ 1 & 1 & -1 & 3 & -3 \\ 2 & 2 & -3 & -1 & -2 \\ 5 & 1 & -5 & 2 & -10 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$, $x = 1, y = 2, z = 3, w = -1$.

(b) $\begin{pmatrix} 3 & -2 & 1 & 5 \\ 1 & 1 & -1 & 7 \\ 9 & -1 & -1 & 9 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 & -\frac{1}{5} & 0 \\ 0 & 1 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, no solution.

15. (a) $x = 7 + \dfrac{\ln 4}{\ln 3} \approx 8.2619$.
(b) $x = \ln 6 \approx 1.7918$.

16. (a) 5 tons. (b) $y = 2(x-2)^2 - 3$.

17. (a) $x \neq 7$, (b) $[0,2] \cup [3,\infty)$, (c) All x.

18. $3x - 5 + \dfrac{11x - 3}{x^2 + x - 1}$.

19. $\dfrac{(2x+3)(x-3)}{3(x+2)}$, $x \neq 0, -3$.

20. $(x, y) = (2, 4)$ or $(-\frac{3}{4}, -\frac{23}{8})$.

21. $x \neq 0$, $x \geq 3$. $f \circ g = \dfrac{3}{\sqrt{x-3}}$, $g \circ f = \sqrt{\dfrac{3}{x} - 3}$.

22. (a) No inverse (graph fails horiz. line test), (b) $f^{-1}(x) = \dfrac{3}{x} + 5$, (c) $f^{-1}(x) = \dfrac{1}{2}(x^3 - 1)$.