Review Questions for Exam 3

Disclaimer: This is not a *practice exam.* It is meant to reflect the sort of material you will be expected to know on the exam, not to indicate the actual questions or length of the exam.

1. $f(x) = -2x^2 + 4x + 3$.
(a) Find the axis of symmetry and vertex.
(b) Find the intercepts.
(c) Sketch the graph of f and write f in standard form.

2. Find the quotient and remainder when $2x^4 - 7x^3 - 3x + 2$ is divided by $x^2 - 3x + 1$.

3. Give a polynomial of degree 4 with real coefficients, a root of multiplicity 2 at -1 and a root at $3 + \sqrt{2}\, i$.

4. $f(x) = \dfrac{3x^2 - x - 2}{x^2 - 5x + 6}$, $g(x) = \dfrac{x^2}{x^4 + 1}$, $h(x) = \dfrac{x^3 + 2x^2 + 1}{x^2 - 2x + 1}$.
Identify the asymptotes (vertical, horizontal and slant) and test for symmetry

5. $f(x) = \dfrac{x^3 - 4x}{x^2 - 1}$.

Find the (a) intercepts, (b) asymptotes, (c) symmetry. (d) Where is f positive? (e) Where is f negative? Use this information to sketch the graph.

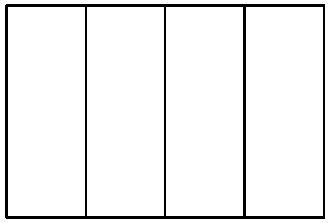
6. $f(x) = x^4 + 3x^3 - x - 3$.
(a) Find the rational roots of f.
(b) Factor f over the reals.
(c) Factor f into linear factors (use complex numbers as necessary).

7. (a) Find the rational roots of $f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$.
(b) Find the remaining roots.

8. $f(x) = x^4 + 5x^3 + 6x^2$
(a) Find the zeros and their multiplicities.
(b) Find where f is positive.
(c) Determine the behavior of f as $x \to \infty$ and as $x \to -\infty$.
(d) Sketch the graph of f.

9.(i) A company sells 5000 lbs of its product if it charges \$10 per pound. For every \$1 increase in price the demand will decrease by 100 lbs. Suppose that the price is increased by $\$x$.
(a) Write the new price and demand in terms of x. Write the revenue in terms of x.
(b) What choice of x maximizes the revenue?

(ii) A farmer has 1000ft of fencing to enclose four adjacent corrals as shown. What width x maximizes the enclosed area?



10. Find the constant c such that

$$\frac{x^3 - 2x^2 + cx - c}{x - 3} = \text{ polynomial } + \frac{3}{x - 3}$$

(i.e. the remainder when $x^3 - 2x^2 + cx - c$ is divided by $(x - 3)$ is 3).

11. Give the equation of the parabola with vertex $(-3, 11)$ which passes through $(-2, 7)$.

Solutions

1. (a) $x = 1$, $(1, 5)$.
(b) x-intercepts: $(1 + \frac{1}{2}\sqrt{10}, 0)$, $(1 - \frac{1}{2}\sqrt{10}, 0)$; y-intercept: $(0, 3)$.
(c) $y = -2(x - 1)^2 + 5$.

2. Quotient $2x^2 - x - 5$, Remainder $-17x + 7$.

3. e.g. $(x + 1)^2(x - 3 - \sqrt{2}i)(x - 3 + \sqrt{2}i) = x^4 - 4x^3 + 16x + 11$.

4. f : $x = 3, x = 2$, $y = 3$, no slant, no symmetry.
g : No vert., $y = 0$, no slant, even.
h : $x = 1$, no horiz., $y = x + 4$, no symmetry.

5. (a) $(0, 0)$, $(2, 0)$, $(-2, 0)$,
(b) vert. $x = 1, x = -1$, slant $y = x$,
(c) odd.
(d) $(-2, -1) \cup (0, 1) \cup (2, \infty)$.
(e) $(-\infty, -2) \cup (-1, 0) \cup (1, 2)$.

6. (a) $x = -3, 1$.
(b) $f = (x + 3)(x - 1)(x^2 + x + 1)$.
(c) $f = (x + 3)(x - 1)(x - \frac{1}{2}(-1 + \sqrt{3}i))(x - \frac{1}{2}(-1 - \sqrt{3}i))$.

7. (a) $x = -1, \frac{3}{2}$.
(b) $x = \sqrt{5}, -\sqrt{5}$.

8. (a) $x = 0$ (mult 2), $x = -2$ (mult 1), $x = -3$ (mult 1).
(b) $(-\infty, -3) \cup (-2, 0) \cup (0, \infty)$.
(c) $+\infty, +\infty$.

9.(i) (a) $p = 10 + x$, $d = 5000 - 100x$, $R = -100x^2 + 4000x + 50000$. (b) $x = \$20$.
(ii) $x = 100$ft.

10. $c = -3$.

11. $y = -4(x + 3)^2 + 11$.