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MATH 100 College Algebra – **Exam I**

7:30–8:20pm, Tuesday February 12, 2002

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1. (8 points) Simplify, using only positive exponents (you can assume that $x, y, z \neq 0$):

$$\frac{(xy^2)^3\, y\, z^2}{(xy^3z)^3} = \frac{x^3y^6yz^2}{x^3y^9z^3} = \frac{x^3y^7z^2}{x^3y^9z^3} = \frac{1}{y^2z}.$$

2. (10 points)

(a) Write $\dfrac{\sqrt{x^3}}{\sqrt[3]{x^2}}$ as a rational power of x (you can assume x is positive here).

$$\frac{\sqrt{x^3}}{\sqrt[3]{x^2}} = \frac{x^{3/2}}{x^{2/3}} = x^{\frac{3}{2}-\frac{2}{3}} = x^{\frac{5}{6}}.$$

(b) Simplify $\sqrt{9y^6}$ by extracting squares (y could be negative here).

$$\sqrt{9y^6} = \sqrt{(3y^3)^2} = |3y^3| = 3|y|^3.$$

3. (6 points) Expand $(x-2)(x^2+2x+4)$ to give a polynomial in standard form.

$$(x-2)(x^2+2x+4) = x(x^2+2x+4) - 2(x^2+2x+4) = x^3+2x^2+4x-2x^2-4x-8 = x^3-8.$$

4. (8 points) Simplify $\dfrac{3-x}{x^2-4x+3}$.

$$\frac{3-x}{x^2-4x+3} = \frac{3-x}{(x-3)(x-1)} = \frac{-(x-3)}{(x-3)(x-1)} = \frac{-1}{x-1}, \qquad (x \neq 3)$$

5. (10 points) Factor the following as far as possible:

(a) $2x^2 - 7x + 3 = (2x-1)(x-3)$.

(b) $4x^2(x-5) - 9(x-5) = (4x^2-9)(x-5) = (2x+3)(2x-3)(x-5)$.

6. (8 points) Solve the linear equation $\sqrt{3}(x+3) - 5 = 2(\sqrt{3} - x)$ for x. Give the exact value of x (a calculator-produced decimal will not earn full credit).
$\sqrt{3}(x+3) - 5 = 2(\sqrt{3} - x)$,
$\sqrt{3}x + 3\sqrt{3} - 5 = 2\sqrt{3} - 2x$, (distribute $\sqrt{3}$ and 2.)
$\sqrt{3}x + 2x = 2\sqrt{3} - 3\sqrt{3} + 5$, (move the terms to the other side and change the sign.)
$(\sqrt{3}+2)x = 5 - \sqrt{3}$. (combine the like terms.) $x = \frac{5-\sqrt{3}}{2+\sqrt{3}}$ (divide $2+\sqrt{3}$.)

7. (8 points) Solve $\dfrac{7}{x} = \dfrac{3}{x-1} - \dfrac{2}{x(x-1)}$ for x.

Common denominator is $x(x-1)$. Multiply the common denominator to both sides:
$7(x-1) = 3x - 2$
$7x - 7 = 3x - 2$ $7x - 3x = 7 - 2 = 5$ $4x = 5$ $x = \frac{5}{4}$.
Check: since $\frac{5}{4} \neq 0, 1$, $x = \frac{5}{4}$ is the solution.

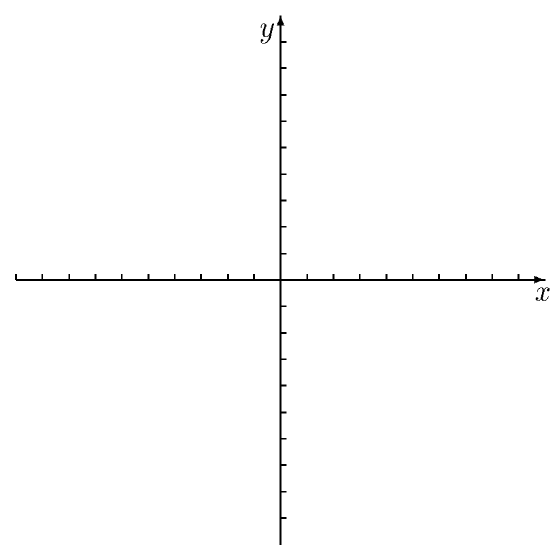
8. (8 points) Use your calculator to graph

$$y = x^3 - 4x + 5.$$

Find the coordinates of the intercepts correct to three decimal places:

x-intercept=(-2.457,0)

y-intercept=(0, 5)



9. (8 points) Perform the operation and simplify $\dfrac{4}{x^2-4}+\dfrac{1}{x+2}$.

$\dfrac{4}{x^2-4}+\dfrac{1}{x+2}=\dfrac{4}{(x-2)(x+2)}+\dfrac{(x-2)}{(x+2)(x-2)}=\dfrac{4+x-2}{(x-2)(x+2)}$
$\dfrac{x+2}{(x+2)(x-2)}=\dfrac{1}{x-2}, \quad (x \neq -2).$

10. (8 points) Find all real values of x (if any) that satisfy the equation

$$x^2 = 3 - 5x.$$

Give exact values (not calculator-produced decimal approximations).

The quadratic equation is $x^2+5x-3=$. Using the quadratic formula: $x = \dfrac{-5\pm\sqrt{5^2-4\cdot 1\cdot(-3)}}{2}=\dfrac{-5\pm\sqrt{37}}{2}$. The two solutions are $x=\dfrac{-5+\sqrt{37}}{2}$ and $x=\dfrac{-5-\sqrt{37}}{2}$.

11. (10 points) A total of \$2000 was deposited for a year in two accounts. At the end of the year one of the accounts had earned 4% interest while the other had earned 5% interest. The total interest earned was \$85. Find the amount, x, deposited in the account paying 4%.

Let x be the amount deposited in the 4% amount. Then $2000-x$ are deposited in the account with 5% account. The interest one year from the first account is $0.04x$ and the interests from the second account is $0.05(2000-x)$. So the total interests for the year is

$$0.04x + 0.05(2000 - x) = 85$$

Solve the equation for x
$0.04x - 0.05x + 100 = 85$
$-0.01x = -15$
$x = -15/(-0.01) = 1500$. Thus \$1500 is deposited to the account with 4% interests.

12. (8 points)
(a) Find the distance between the points $(2,-3)$ and $(5,1)$.
$d=\sqrt{(2-5)^2+(-3-1)^2}=\sqrt{(-3)^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$. (b) Give the standard equation of the circle with center $(2,-3)$ which passes through $(5,1)$.

The equation of circle with center at $(2,-3)$ is $(x-2)^2+(y-(-3))^2=r^2$. We are to find what the radius r is.

Method 1. Since the point $(5,1)$ is on the circle and the radius is the distance from between $(2,-3)$ and $(5,1)$, which is computed to be 5 above. Thus the equation is $(x-2)^2+(y+3)^2=5^2$.

Method 2. Since the point $(5,1)$ is on the circle. So it is a solution of the equation. Plug the point $(5,1)$ into the equation to get

$$r^2 = (5-2)^2 + (1-(-3))^2 = 3^2 + 4^2 = 25.$$

The equation is $(x-2)^2+(x+3)^2=25$.