Math 100 Exam III

7:30pm–8:30pm, Tuesday November 6, 2001

Recitation Instructor:____________________

Recitation Day/Time:____________________ **Name:**____________________________

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** Use back pages as scratch paper. Do problems at spaces provided.

Total/100	Page1/30	Page2/30	Page3/40

1. (10pts) For the quadratic function $f(x) = 2x^2 + 8x + 7$

(a) determine the vertex of the parabola;

$h = -\dfrac{b}{2a} = -\dfrac{8}{2\cdot 2} = -2$, $k = f(h) = f(-2) = 2\cdot(-2)^2 + 8\cdot(-2) + 7 = -1$.

The vertex is $(-2, -1)$.

(b) determine all x-intercepts (with exact values).

$f(x)$ in *Standard Form*;

$f(x) = a(x-h)^2 + k = 2(x-(-2))^2 + (-1) = 2(x+2)^2 - 1$

Solve $f(x) = 0$. $\quad 2(x+2)^2 - 1 = 0, \quad 2(x+2)^2 = 1, \quad (x+2)^2 = \frac{1}{2}, \quad x + 2 = \pm\frac{1}{\sqrt{2}}$, $x = -2 \pm \frac{\sqrt{2}}{2}$.

There are two intersects, which are $(-2 - \frac{\sqrt{2}}{2}, 0)$ and $(-2 + \frac{\sqrt{2}}{2}, 0)$.

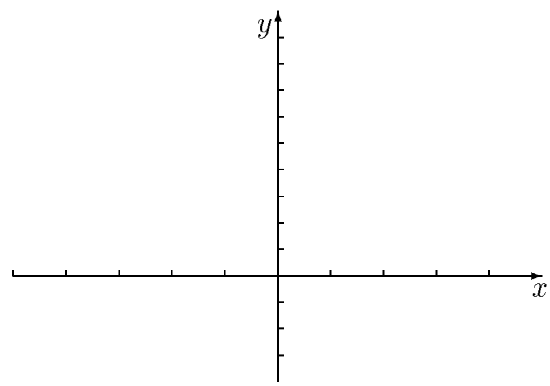
2. Graph the function $f(x) = x^4 - 5x^2 + 4$ with the following steps:

(a) (3pts) Find all zeros of the function.

$f(x) = x^4 - 5x^2 + 4 = (x^2-1)(x^2-4)$

$= (x-1)(x+1)(x-2)(x+2) = 0$

All zeros of $f(x)$ are $-2, \quad -1, \quad 1, \quad 2$.



(b). (3pts) Plot the following points.

x	-2.5	-1.5	0	1.5	2.5
$f(x)$	11.81	-2.19	4	-2.19	11.91

(c) (4pts) Graph the function to indicate the properties in (a) and (b) and the left-hand and right-hand behavior.

3. (a) (6pts) Use long division to find the quotient and remainder when $2x^4 - 4x^3 - 5x^2 + 3x - 2$ is divided by $x^2 - 2x - 3$.

$$\begin{array}{rrrrrrr}
 & & & 2x^2 & & +1 \\ \cline{2-6}
x^2-2x-3 \;) & 2x^4 & -4x^3 & -5x^2 & +3x & -2 \\
 & 2x^4 & -4x^3 & -6x^2 & & \\ \cline{2-6}
 & & & x^2 & +3x & -2 \\
 & & & x^2 & -2x & -3 \\ \cline{4-6}
 & & & & 5x & +1
\end{array}$$

quotient $q(x) = 2x^2 + 1$,
remainder $r(x) = 5x + 1$.

(b) (4pts) Write the following fraction as a sum of a polynomial and a proper fraction.

$$\frac{2x^4 - 4x^3 - 5x^2 + 3x - 2}{x^2 - 2x - 3} = 2x^2 + 1 + \frac{5x+1}{x^2 - 2x - 3}$$

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4. (10pts) For the polynomial $f(x) = x^4 - x^3 + x^2 - 3x - 6$,

(a). list all possible rational zeros using the rational zero test and find all rational zeros;

Possible rational zeros are $\frac{p}{q}$ with p being a factor of -6 and q a factor 1.

The list of possible rational zeros is: $\quad 1,\ -1,\ 2,\ -2,\ 3,\ -3,\ 6,\ -6$

Evaluate $f(x)$ at all values of x in the list and only $x = -1, 2$ are rational zeros.

(b). find all remaining zeros (in addition to those in (a)) of $f(x)$ including complex zeros as well.

Use synthetic division by factors $(x+1)$ and $(x-2)$

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array} \qquad \begin{array}{r|rrrr} 2 & 1 & -2 & 3 & -6 \\ & & 2 & 0 & 6 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

Then $f(x) = (x+1)(x-2)(x^2+3)$ and all zeros are: $-1,\ 2\ \sqrt{3}i,\ -\sqrt{3}i$.

5. (10pts) For each of the following functions, find its domain and all vertical, horizontal, and slant asymptotes (write down their equations).

	domain	v. asymp.	h. asymp.	slant asymp.
(a). $f(x) = \frac{5}{(x-3)^3}$	all real $x \neq 3$	$x = 3$	$y = 0$	No
(b). $f(x) = \frac{x^2 - 2x}{x+1}$	all real $x \neq -1$	$x = -1$	No	$y = x - 3$

$$\begin{array}{r|rrr} -1 & 1 & -2 & 0 \\ & & -1 & 3 \\ \hline & 1 & -3 & 3 \end{array}$$

$$f(x) = \frac{x^2 - 2x}{x+1} = (x-3) + \frac{3}{x+1}$$

6. (10pts) Graph the function $f(x) = \dfrac{2(x^2-9)}{x^2-4}$ and provide the following information:

(a). y-intercepts:__________ $\frac{9}{2} = 4.5$

(b). x-intercepts:__________ $(-3, 0),\ (3, 0)$

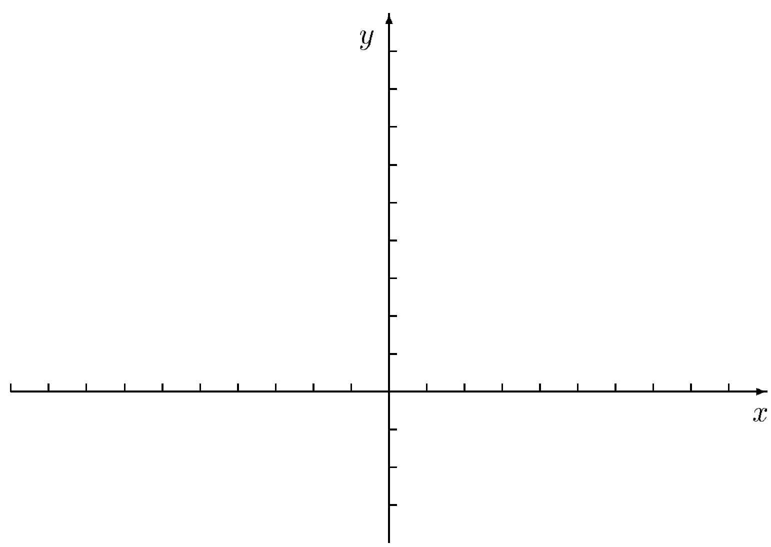
(c). Vertical asymptotes:__ $x = -2,\ x = 2$

(d). Horizontal asymptotes:______ $y = 2$

(e). Symmetry:________Sym. in y-axis

(e). Values of the function:

x	0.5	1.5	2.5	7
$f(x)$	4,67	7.71	−2.44	1.78



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7. (10pts) Write the partial fraction decomposition for $\frac{x+7}{x^2-x-6}$.
$x^2-x-6=(x-3)(x+2)$.
$\frac{x+7}{x^2-x-6}=\frac{A}{x-3}+\frac{B}{x+2}=\frac{A(x+2)+B(x-3)}{(x-3)(x+2)}$. $\quad x+7=A(x+2)+B(x-3)$.
Set $x=3$ to get $3+7=A(3+2)+B(3-3)=5A$, $A=2$
Set $x=-2$ to get $-2+7=A(-2+2)+B(-2-3)=-5B$, Thus $B=-1$.
$\frac{x+7}{x^2-x-6}=\frac{2}{x-3}+\frac{-1}{x+2}$

8. (10pts) Find the number of unites old that produces a maximum revenue $R=900x-0.1x^2$, where R is the total revenue (in British Pounds) and x is the number of units sold.

$R=900x-0.1x^2=-0.1(x^2-9000x)=-0.1(x-9000x+(9000/2)^2)+0.1\cdot(9000/2)^2=-0.1(x-4500)^2+202500$ is a quadratic function opening downward.
The x coordinate of the vertex is $h=-\frac{b}{2a}=-\frac{900}{-0.2}=4500$.
When $x=4500$ the revenue $R=202500$ British Pounds is maximal.

9. (10pts) Convert the logarithm equation to an equivalent exponential equation, and vice versa.
(a). $\log_{32}(4)=\frac{2}{5}$, the equivalent exponential equation is
$4=32^{\frac{2}{5}}$

(b). $9^{\frac{3}{2}}=27$, the equivalent logarithm equation is $\log_9(27)=\frac{3}{2}$.

10 (10pts) Given that x^2+4 is a factor, factor the polynomial $x^4-3x^3-x^2-12x-20$ as a product of linear factors including complex coefficients.
Using long division to get $x^4-3x^3-x^2-12x-20=(x^2+4)(x^2-3x-5)$.
$x^2+4=0$ with solution $x=\pm 2i$.
$x^2-3x-5=0$ with solution $x=\frac{3\pm\sqrt{9-4\cdot(-5)}}{2}=\frac{3}{2}\pm\frac{\sqrt{29}}{2}$.
$x^4-3x^3-x^2-12x-20=(x-2i)(x+2i)(x-(\frac{3}{2}+\frac{\sqrt{29}}{2}))(x-(\frac{3}{2}-\frac{\sqrt{29}}{2}))$.

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