Math 100 Exam II

7:30pm–8:30pm, Tuesday October 9, 2001

Recitation Instructor:____________________

Recitation Day/Time:____________________ **Name:Answer Key**

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as a scratch paper. Do the problems at the space provided.

Total/100	#pg1/30	#pg2/40	#pg3/30

1. (10pts) Write the complex numbers in standard form:

(a) $(3+2i)^2 = 3^2 + 2\cdot 3\cdot 2i + (2i)^2 = 9 + 12i + 4i^2 = 9 + 12i - 4 = 5 + 12i$

(b) $\dfrac{2+3i}{3+2i} = \dfrac{(2+3i)(3-2i)}{(3+2i)(3-2i)} = \dfrac{6-4i+9i-6i^2}{9+4}$

$= \dfrac{12+5i}{13} = \dfrac{12}{13} + \dfrac{5}{13}i$

2. (10pts) Find all (real and complex) solutions of the following equations.

(a) $x^3 - 3x^2 + 3x - 9 = 0$

$x^3 - 3x^2 + 3x - 9 = x^2(x-3) + 3(x-3) = (x^2+3)(x-3)$,

$(x^2+3)(x-3) = 0$,

$x^2 = -3, \qquad x = 3,$

$x = \sqrt{3}\, i, \quad -\sqrt{3}\, i, \quad 3.$

(b) $\dfrac{2}{x} = \dfrac{3}{x-2} + 1$

Multiply LCD $x(x-2)$ on both sides to get $2(x-2) = 3x + x(x-2)$

$2x - 4 = 3x + x^2 - 2x, \quad x^2 - x + 4 = 0.$ Quadratic formula: $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4\cdot 1\cdot 4}}{2} = \dfrac{1}{2} \pm \dfrac{\sqrt{15}}{2}\, i.$

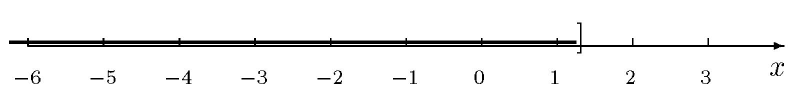
$x = \dfrac{1}{2} + \dfrac{\sqrt{15}}{2}\, i, \qquad \dfrac{1}{2} - \dfrac{\sqrt{15}}{2}\, i.$

3. (10pts) Solve the Inequalities and graph your solutions:

(a) $1 - 3x \geq x - 4$

$5 \geq 4x$,

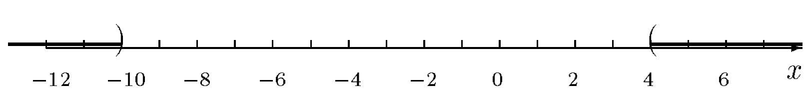
$\frac{5}{4} \geq x, \qquad x \leq 1\frac{1}{4}.$



(b) $|x+3| > 7$

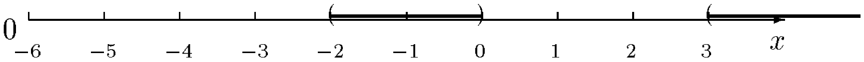
$x + 3 > 7$, or $x + 3 < -7$

$x > 4$, or $x < -10$.



4. (10pts) Solve the inequality $x^3 - x^2 - 6x > 0$ and graph your solutions.

$x^3 - x^2 - 6x = x(x^2 - x6) = x(x-3)(x+2) = 0$



critical numbers are $-2,\ 0,\ 3$.

Intervals: $(-\infty, -2)$, $(-2, 0)$, $(0, 3)$, $(3, \infty)$

Test values	-3	-1	1	4
$x^3 - x^2 - 6x$	-18	4	-6	24

Intervals such that $x^3 - x^2 - 6x > 0$ are $(-2, 0)$ and $(3, \infty)$

5. (10pts) Write down the equation of the straight line passing through two points $(1, 2)$ and $(3, 2)$. Then compute the y-intercept and slope of the line.

Slope=0

Slope $= \dfrac{2-2}{3-1} = 0$.

y-intercept=(0,2)

The equation of the line is $y - 2 = 0 \cdot (x - 1)$. Thus $y = 2$. $y = 2$ when $x = 0$.

6. (10pts) Your salary was \$28,500 in year 1997 and \$32,900 in 1999. What will be your salary in year 2005 if your salary follows a linear growth.

Slope $= \dfrac{32900 - 28500}{1999 - 1997} = \dfrac{4400}{2} = 2200$.

Let y be the salary in the year x. Then the linear equation is $y - 28500 = 2200(x - 1997)$

$y = 2200(x - 1997) + 28500$.

When $x = 2005$, the salary is $y = 2200(2005 - 1997) + 28500 = 17600 + 28500 = 46100$.

The salary in year 2005 is \$46100

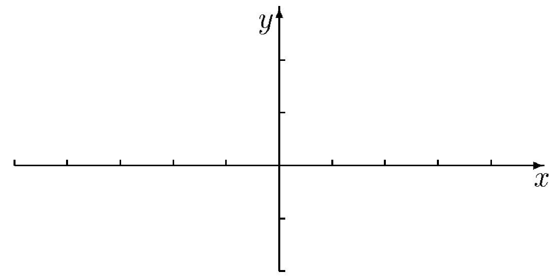
7. (10pts) Let $g(x) = -x^2 + 4x + 1$. Evaluate and simplify

(a) $g(-2) = -(-2)^2 + 4 \cdot (-2) + 1 = -4 - 8 + 1 = -11$.

(b) $g(h+2) = -(h+2)^2 + 4(h+2) + 1 = -(h^2 + 4h + 4) + 4h + 8 + 1 = -h^2 + 5$.

8. (10pts) Let $f(x) = x^3 - 3x^2 - x + 1$. Use your calculator to

(a). (2pts) Graph the function. (copy the graph from your calculator carefully and write down your window settings.)



Xmin=-3, Xmax=5, Ymin=-6, Ymax=3

(b). Find the relative minimum and relative maximum (both x and y-coordinates) (approximate to 3rd decimal place).
Minimum: $(2.155, -5.079)$
Maximum: $(-.1547, 1.079)$

(c). List the intervals of increasing and the intervals of decreasing.

Intervals of increasing:
$(-\infty, -.1547)$ and $(2.155, \infty)$

Intervals of decreasing:
$(-.1547, 2.155)$

9. (10pts) Given functions $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$,
(a).(2pts) Find the domains of functions $f(x)$ and $g(x)$;

$f(x)$
All real numbers x.

$g(x)$
$x \geq 0$.

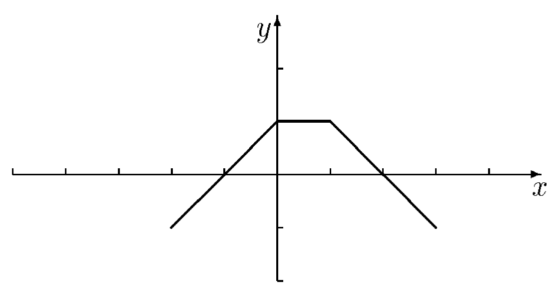
(b). Find $f \circ g$ and its domain;
$(f \circ g)(x) = f(g(x)) = (g(x))^2 + 1$
$= (\sqrt{x})^2 + 1 = x + 1$.

Domain: $x \geq 0$

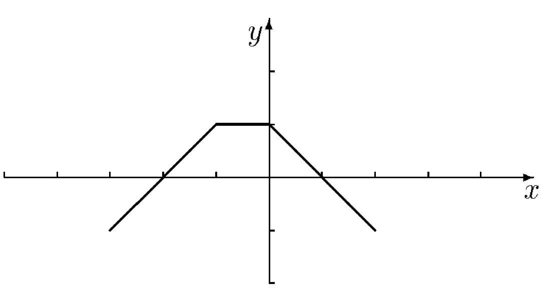
(c). Find $g \circ f$ and its domain;
$(g \circ f)(x) = g(f(x)) = \sqrt{f(x)}$
$= \sqrt{x^2 + 1}$.

Domain: All real values x.

10 (10pts) The graph of a function $f(x)$ is on the right. Graph the given functions.



(a). $y = f(-x)$



(b). $y = 2f(x)$