Math 100 Exam I

7:30pm–8:30pm, Tuesday September 11, 2001

Recitation Instructor:____________________

Recitation Day/Time:____________________ **Name:**____________________________

No books or notes are allowed. Please read the problems carefully and do **all** you are asked to do. **You must show your work!** You can use the back page as scratch paper and do the problems at the space provided.

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1. (10pts) Simplify using rational exponents only (x is positive and y could be any real number).
$\sqrt[4]{16x^3y^2} =$

$$\begin{aligned} &= (16x^3y^2)^{1/4} \\ &= 16^{1/4}(x^3)^{1/4}(y^2)^{1/4} \\ &= 2x^{3/4}((y^2)^{1/2})^{1/2} = 2x^{3/4}|y|^{1/2} \end{aligned}$$

2. (10pts) Simplify (x could be any nonzero real number)
$\frac{(2x^2)^{3/2}}{2^{1/2}x^4} =$

$$\begin{aligned} \frac{(2x^2)^{3/2}}{2^{1/2}x^4} &= \frac{2^{3/2}(x^2)^{3/2}}{2^{1/2}(x^2)^2} \\ &= 2^{3/2-1/2}(x^2)^{3/2-2} = 2^1(x^2)^{-1/2} \\ &= \frac{2}{(x^2)^{1/2}} = \frac{2}{|x|} \end{aligned}$$

3. (10pts) Multiply the polynomial
$(x^2 - x + 1)(x^2 + x + 1) =$

$$\begin{aligned} (x^2 - x + 1)(x^2 + x + 1) &= (x^2 + 1 - x)(x^2 + 1 + x) \\ &= ((x^2 + 1) - x)((x^2 + 1) + x) \\ &= (x^2 + 1)^2 - x^2 \\ &= (x^2)^2 + 2x^2 + 1 - x^2 \\ &= x^4 + x^2 + 1 \end{aligned}$$

4. (10pts) Factor
(a). $2x^2 + x - 15 =$

$2x^2 + x - 15 = (2x - 5)(x + 3)$
since $(2x - 5)(x + 3) = 2x \cdot x + 2x \cdot 3 - 5 \cdot x - 5 \cdot 3 = 2x^2 + x - 15.$

(b). $x^2 - 10x + 25 =$

$x^2 - 10x + 25 = x^2 - 2 \cdot x \cdot 5 + 5^2 = (x - 5)^2$

5. (10pts) Simplify $\dfrac{x^3 + 5x^2 + 6x}{x^2 - 4} =$

$$\begin{aligned} \frac{x^3 + 5x^2 + 6x}{x^2 - 4} &= \frac{x(x^2 + 5x + 6)}{(x - 2)(x + 2)} \\ &= \frac{x(x + 3)(x + 2)}{(x - 2)(x + 2)} = \frac{x(x + 3)}{x - 2} \qquad (x \neq -2) \end{aligned}$$

6. (10pts) Solve for x and check your solution. $\dfrac{3}{x(x+1)} + \dfrac{7}{x+1} = \dfrac{3}{x}$.

$\dfrac{3}{x(x+1)} + \dfrac{7 \cdot x}{(x+1) \cdot x} = \dfrac{3(x+1)}{x \cdot (x+1)}$
Multiply the Least Common Denominator $x(x + 1)$ to both sides:
$3 + 7x = 3(x + 1) = 3x + 3 \qquad (x \neq 0, -1)$
$7x - 3x = 3 - 3, \quad 4x = 0, \quad x = 0.$
Since the denominator will zero when $x = 0$, the equation has NO SOLUTION!

7. (10pts) An auto dealer has \$600,000 inventory in two types of cars, compact and mid-size. The profit on compact cars is 24% and the profit on mid-size cars is 28%. The profit for the entire stock was 25%. How much was invested in each type of cars?

Verbal Model: investment $\times$ percent = profit.
Let x be the amount invested in compact cars. Then $600,000 - x$ is invested to mid-size cars.
Profit from compact cars $= 0.24x$, Profit from mid-size cars $= 0.28(600, -x)$.
Total profit: $0.25 \times 600,000 = 0.24x + 0.28(600,000 - x)$.
$0.25 \times 600,000 = 0.24x - 0.28x + 0.28 \times 600,000 = -0.04x + 0.28 \times 600,000.$
$0.04x = 0.28 \times 600,000 - 0.25 \times 600,000 = 0.03 \times 600,000.$
$x = 0.03 \times 600,000/0.04 = 450,000, \quad 600,000 - x = 150,000.$
The investment to compact car is \$450,000 and to mid-size car is \$150,000.

8. (10pts) Solve the equation $x^2+3x=9$ for *exact* solution.
Use quadratic formula: simplify the equation to general form:
$x^2+3x-9=0. \quad a=1, \quad b=3, \quad c=-9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3\pm\sqrt{9-4(-9)}}{2}=\frac{-3\pm\sqrt{45}}{2}=\frac{-3\pm3\sqrt{5}}{2}$$

$x=\frac{-3+3\sqrt{5}}{2}$ and $x=\frac{-3-3\sqrt{5}}{2}$.

9. (10pts) A bedroom is 3 feet longer than it is wide and has an area of 154 square feet. Find the dimensions of the room.

Verbal Model: width $\times$ length $=$ area.
Let x be the width of the room. Then $x+3$ is the length of the room.
$x(x+3)=154, \quad x^2+3x-154=0$. Use the quadratic formula to solve the equation:

$$x=\frac{-3\pm\sqrt{9-4(-154)}}{2}=\frac{-3\pm\sqrt{9+616}}{2}=\frac{-3\pm\sqrt{625}}{2}=\frac{-3\pm25}{2}.$$

$x=\frac{-3+25}{2}=11$ and $x=\frac{-3-25}{2}=-14$. Since the room should have positive dimension, $x=11$ is the width of the room and $11+3=14$ is the length of the room. The dimensions of the room are 11 feet wide and 14 feet long.

10 (10pts) Perform the operation and simply the fractions:

(a) $\frac{2}{x^2-x-2}+\frac{10}{x^2+2x-8}$

$$=\frac{2}{(x-2)(x+1)}+\frac{10}{(x+4)(x-2)}=\frac{2\cdot(x+4)}{(x-2)(x+1)\cdot(x+4)}+\frac{10\cdot(x+1)}{(x+4)(x-2)\cdot(x+1)}$$
$$=\frac{2x+8+10x+10}{(x+4)(x-2)(x+1)}=\frac{12x+18}{(x+4)(x-2)(x+1)}=\frac{6(2x+3)}{(x+4)(x-2)(x+1)}.$$

(b) $\dfrac{\left(\dfrac{x^2-1}{x}\right)}{\left[\dfrac{(x-1)^2}{x}\right]}$

$$=\left(\frac{x^2-1}{x}\right)\div\left[\frac{(x-1)^2}{x}\right]=\frac{(x-1)(x+1)}{x}\times\frac{x}{(x-1)^2}$$
$$=\frac{(x+1)}{1}\times\frac{1}{(x-1)}=\frac{x+1}{x-1}. \quad (x\neq0)$$