Introduction to Geometric Measure Theory

MATH 992, Fall 2001
Marianne Korten

In this course we will introduce the concepts of Hausdorff measures and dimensions. These "lower dimensional" measures allow to measure certain "very small" sets, and arise naturally in variational calculus, geometric analysis, harmonic analysis, PDE and potential theory. We will start by reviewing covering lemmas and differentiation. We will then define Hausdorff measures and dimensions, study their properties and typical examples. We will discuss Lipschitz mappings, jacobians, and differentiability. Then we will prove the area and co-area formulas (generalized "change of variables" formulas) for Lipschitz mappings. These formulas are basic and powerful tools that involve Hausdorff measures.

Prerequisites: Math 822 - Real Analysis.

Text: No text will be required.

We will use the following references (in this order):

1. Gaëlle Chabod and Carine Krähenbül, *Mesure et dimension de Hausdorff,* Master's thesis, Université de Franche-Comté, July 1999.

2. C. L. Evans and R. Gariepy, *Measure theory and fine properties of functions,* CRC Press 1992.

3. F. Morgan, *Geometric measure theory: A beginner's guide,* Academic Press, 1995.

4. P. Mattila, *Geometry of sets and measures in euclidean spaces,* Cambridge Univ. Press 1995.

5. L. Simon, *Lectures on geometric measure theory,* Proc. Centre Math. Analysis ANU, Canberra, 1983.

6. W. Ziemer, *Weakly differentiable functions,* Springer, 1989.

7. E. Giusti, *Minimal surfaces and functions of bounded variation,* Birkhäuser, 1985.

8. L. Carleson, *Selected problems on exceptional sets,* Van Nostrand, 1967.