Math 997, Fall 2008, Algebraic Number Theory

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I) Preliminaries.
  1) Chinese Remainder Theorem for commutative rings.
  2) Modules over commutative rings: Free modules, rank.
  3) Modules over PIDs: Invariant factors, fundamental theorem, torsion.
  4) Lattices and discrete subgroups.
  5) Geometry of Numbers: Minkowski's Theorem, Lagrange's 4-squares theorem.

II) Algebraic Number Fields.
  1) Rings of integers in algebraic extensions of Q.
  2) Characteristic polynomial and minimal polynomial.
  3) Norm, Trace and Discriminant.
  4) Integral extensions of rings.
  5) Integrally closed rings.
  6) Cyclotomic fields.

III) Noetherian Rings and Dedekind Domains.
  1) Prime ideals, maximal ideals and fractional ideals.
  2) Rings of integers are Dedekind domains.
  3) Unique factorization of ideals in Dedekind domains.
  4) Finiteness of ideal class group.
  5) Dirichlet's class number formula for quadratic fields.

IV) Units in Rings of Integers of Number Fields.
  1) Dirichlet's structure theorem for the group of units.
  2) Pell's equation.
  3) Fermat's Last Theorem in the case of UFDs.

V) Factorization of Prime Ideals in Extension Fields.
  1) Splitting of prime ideals.
  2) Localization of a Dedekind domain is a D.V.R..
  3) Degree of inertia and ramification.
  4) p ramifies iff p divides the discriminant.

VI) Galois Extensions.
  1) Decomposition subgroup and Frobenius automorphism.
  2) Cyclotomic fields.
  3) Quadratic reciprocity.

References:

P. Samuel, *Algebraic Theory of Numbers,* Hermann (1970).
H. Pollard, *The Theory of Algebraic Numbers,* Dover (1998).
E.Hecke, *Lectures on the Theory of Algebraic Numbers,* Springer-Verlag (1981).
Z.I. Borevich and I.R. Shafarevich, *Number Theory,* Academic Press (1966).
G. Janusz, *Algebraic Number Fields,* Academic Press (1973).
S. Alaca and K. Williams, *Introductory Algebraic Number Theory,* Cambridge (2004).

Grading: I plan on giving "brief" monthly homework assignments and having each student in the class give a one day presentation. Let me know if there is a topic you would like to talk about. I also have a number of topics that you (or I) can choose from. There will not be any exams.