Topics in Algebra
MATH 991

Algebraic Deformation Theory

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Fall 2007

Mathematical structures on an given underlying space often organize themselves into families cut out by equations. Deformation theory systematically studies the structures 'nearby' a given mathematical structure in such a space.

As one example, the associative k-algebra structures on a given vector space V over k are described by equations which cut out a subset of $Hom_k(V \otimes V, V)$. In the case of $k = \mathbb{R}$ or $\mathbb{C}$, this set has a natural manifold structure, in other cases it has the natural structure of a scheme over k.

As another, the complex structures on a surface organize themselves into space, called the moduli space (named from the original example of the torus, on which complex structures correspond to the modulus–the other complex number besides 1 spanning the lattice by which $\mathbb{C}$ was quotiented).

Quite remarkably, in both cases, as well as others to be covered in the course, the infinitesimal structure of the space has the same structure: tangent vectors (called first order infinitesimal deformations) can be identified with classes in a cohomology group naturally constructed from the structure at hand (be it associative algebra, Lie algebra, complex manifold, monoidal category, . . .), while higher order jets are cochains in the same cochain group, which must cobound an 'obstruction', a cocycle in the cochain group of one higher dimension.

The course will focus primarily on this infinitesimal theory.

After lecture introducing (or reviewing) sufficient homological algebra for the study of deformation theory, and other background material not covered in the 700-level algebra sequence, we will change from a lecture-only format to the analog of a 'Great Books' format for 20th century mathematics: will read together the seminal papers in the subject, including foundational papers in deformation quantization, the papers of Gerstenhaber developing the deformation theory of associative algebras, and papers of Gerstenhaber and Schack extending the theory to diagrams of associative algebras.

Depending on interest, we will continue either this semester or in the spring with one or more of the following: Kodaira and Spencer's papers on deformations of complex structure, papers of Yetter and Etingoff on the deformation theory of monoidal categories, or the Kontsevich/Soibelman proof of the Deligne conjecture.

Only a 700-level background in algebra will be assumed.

There will be no required text–papers will be put on reserve.

The course will be graded on the basis of attendance and presentations of material from the papers we read.