Course description for MATH 827: Classical and Modern Fourier Analysis I
Fall 2007

NAME OF THE COURSE: Classical and Modern Fourier Analysis I

SEMESTER: Fall 2007

INSTRUCTOR: Prof. Diego Maldonado

CREDITS: 3

PREREQUISITE: MATH 821 Real Analysis

TEXT: There is no textbook required for the course as we will be using our own notes. However, the following texts are relevant to the course and they are recommended reading:

- *Fourier analysis* by Javier Duoandikoetxea (translated and revised by David Cruz-Uribe). American Mathematical Society, 2001.
- *Classical and Modern Fourier Analysis* by Loukas Grafakos. Pearson/Prentice Hall, 2004.
- *Harmonic analysis : real-variable methods, orthogonality, and oscillatory integrals* by Elias M. Stein. Princeton University Press, 1993.

INSTRUCTOR'S WEB PAGE: http://www.math.ksu.edu/∼dmaldona

TARGET AUDIENCE: Graduate students from physics, engineering, and mathematics who have taken a course on Real Analysis.

GOAL: To develop a sophisticated mathematical machinery that is tightly connected to both: *abstract fields* such as harmonic analysis in locally compact groups, representation theory, diffusion semigroups, and functional analysis; and to *applied fields* such as signal/image processing and partial differential equations.

DESCRIPTION OF THE COURSE: This course is the first part of the sequence MATH 827 and MATH 828 (Classical and Modern Fourier Analysis II). Along with standard theorems, the course will include topics on:

(i) Fourier Analysis on the circle: functions as signals, Fourier series, L^2-convergence of the partial sums of the Fourier series, L^p-convergence, criteria for almost everywhere pointwise convergence, Gibss phenomenon.

(ii) Fourier Analysis on the line: The Fourier transforms, the duality smoothness/decay, the uncertainty principle, fractional integrals, filters, connections to fundamental solutions of PDEs.

(iii) Singular integrals of convolution type: The Hilbert transform, analytic signals and instantaneous frequency, connections to harmonic functions, connections to the L^p-convergence of Fourier series. Singular integrals in higher dimensions, the Riesz transforms, applications to the L^p-theory of elliptic PDEs.

(iv) Littlewood-Paley theory: Wavelets, characterizations of functions spaces (Lebesgue spaces, Hardy spaces, Sobolev spaces, Besov spaces, Triebel-Lizorkin spaces), square functions, multipliers.

(v) BMO, Carleson measures: The duality between H^1 and BMO.

(vi) Singular integrals of non-convolution type: General L^p-boundedness theory, applications to the Cauchy integral.

The course will also include historical references, current open problems, and connections of Fourier Analysis with other areas of Mathematics.

For more information, contact Prof. Maldonado at dmaldona@math.ksu.edu