MATH 789
Combinatorics
Summer 2008
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Combinatorics is the discipline of mathematics that deals with counting infinite structures. It has broad applications throughout mathematics, as often problems involving dimensions can be reduced to a combinatorial problem. The subject also includes structural results about finite arrangements (graph theory, combinatorial geometry, and the theory of block designs being chief sub disciplines dealing with such things), and algorithmic optimization problems about finitary structures (for example the famous 'traveling salesman' problem).

This course will focus on enumerative combinatorics, techniques for counting things. The most basic results are doubtless familiar from some undergrad course on basic probability: techniques which suffice to count the number of different full-houses, flushes, straights… in poker.

The course will cover:

- Basic enumeration techniques
- Frequently seen counting function: binomial coefficients, Catalan numbers, Stirling numbers (of both the first and second kind), Bell numbers…
- Partially ordered sets and incidence algebras
- Seive technique: classical inclusion-exclusion and Mobius inversion
- Polya enumeration
- Generating functions in all three flavors: ordinary, exponential and Dirichlet (the last also turns up in number theory, while Hilbert series of graded modules are a special case of the first)

If time permits, a few topics in structural combinatorics may be covered late in the course. The course is suitable for both graduate students and advance undergrads, and may be taken for either graduate or undergraduate credit. Some topics will be reviewed for students who have taken MATH 510 or an equivalent course at another university. There will be plenty of exercises and even take-home exams.

The following are suggested texts, and one or two may be promoted as required texts:

- Aigner, Martin, *A Course in Enumeration*, Springer GTM 238, Springer (2007).
- Aigner, Martin, *Combinatorial Theory*, Grundlehren der mathematischen Wissenschaften 234, Springer (1979).
- Brualdi, Richard A., *Introductory Combinatorics*, 4th ed., Prentice-Hall (2004.) This one is often the required text for MATH 510, so used copies should be easy to come by.
- Lovasz, Laszlo, *Combinatorial Problems and Exercises*, North-Holland (1979). Lots of problems, with hints and solutions.
- Stanley, Richard P., *Enumerative Combinatorics, Volumes I and II.* (The libraries copies have different publishers, not sure who currently has the rights to this set.)
- Wilf, Herbert S., *Generatingfunctionology*, Academic Press (1990). (Or better still, the newer edition.)

I'll be putting the library's copies of the texts on reserve this summer.