KANSAS STATE UNIVERSITY
Department of Mathematics

S. Thomas Parker Mathematical Competition
April 17, 2004

Instructions: Put your name on all papers you use and turn them all in. Try to solve as many problems as you can. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations to help you for your justification, but your explanation should be convincing.

1. A cylinder is inscribed in a given sphere of unit radius. Find the dimensions of the cylinder so that the surface area (including the top and bottom) of the cylinder is maximal.

2. Compute the following limit

$$\lim_{n\to\infty}\left(\frac{1}{n}\prod_{i=1}^{n}(n+i)^{\frac{1}{n}}\right).$$

3. For a triangle ΔABC and a straight line L (in the same plane as ΔABC) intersecting the interior of the triangle ΔABC, let $\Delta A'B'C'$ be the reflection of the triangle ΔABC with respect to the reflection axis L in the plane. Show that one can always choose the line L so that the area of the intersection of ΔABC and $\Delta A'B'C'$ is at least $2/3$ of the area of ΔABC.

4 Alice and Bob are playing a game. Alice thinks of a secret polynomial $p(x)$ with non-negative integer coefficients. Bob chooses an integer a and Alice tells Bob the value $p(a)$. Then Bob chooses an integer b and Alice tells Bob the value $p(b)$. Now Bob can tell Alice the polynomial $p(x)$ correctly. What is Bob's strategy for determining the polynomial $p(x)$?