KANSAS STATE UNIVERSITY
**Department of Mathematics**

# S. Thomas Parker Mathematical Competition
# April 13, 2002

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**Instructions:** Put your name on all papers you use and turn them all in. Try to solve as many problems as you can. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations to help you for your justification, but your explanation should be convincing.

1. Make an accurate sketch of the graph of the equation $x^3 + x^2y + y^2 - 2x = 0$. Indicate all points where there is a vertical tangent and indicate the asymptotic behavior as $x \to \pm\infty$.

2. A donut has an inner diameter $d$ (i.e., the diameter of the hole) and an outer diameter $D$ (i.e., the distance across from outer edge to outer edge). The cross sections of the donut are circular disks. Derive a formula for the volume of the donut in terms of $d$ and $D$.

3. Consider a fraction $\frac{a}{n}$ in reduced form with $0 < a < n$ being integers.

   (a) Show that there exists a sequence of integers $a_2, a_3, \ldots, a_n$, with $0 \le a_i < i$ for each $i$, such that
   $$\frac{a}{n} = \frac{a_2}{2!} + \frac{a_3}{3!} + \cdots + \frac{a_n}{n!}.$$

   (b) Show that the sequence $a_2, a_3, \ldots, a_n$ is unique.

4. What sector should be cut out from a disk of radius $R$ so that the remaining sector will make a cone with largest possible volume?