KANSAS STATE UNIVERSITY
Department of Mathematics

S. Thomas Parker Mathematical Competition
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Instructions: Put your name on all papers you use and turn them all in. Try to solve as many problems as you can. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations to help you for your justification, but your explanation should be convincing.

1. A car traveling on a straight line at 100 ft/sec hits its brakes and skids to a stop in 100 ft.

   a) Assuming that the deceleration of the car is constant (during its skid) determine how many seconds it takes for the car to come to a stop after first hitting its brakes.

   b) If instead the deceleration is inversely proportional to the speed that the car is traveling (at each instant, determine the time it takes to stop.

2. For any positive integer n, set $(2n)!! = 2 \cdot 4 \cdot 6 \cdots (2n)$ and $(2n-1)!! = 1 \cdot 3 \cdot 5 \cdots (2n-1)$. Prove or disprove that for any positive integer n,

$$\frac{(2n-1)!!}{(2n)!!} \leq \frac{1}{\sqrt{2n+1}}.$$

3. A 6 ft tall man is walking at a constant speed along a straight line away from a 30 ft high street lamp. At the instant that he is 100 ft from the base of the street lamp, his shadow reaches a vertical wall in front of him and begins to rise at a rate of one foot per second. How many more seconds after this instant does it take for him to reach the wall?

4. An m by n checkerboard consisting of $m \cdot n$ identical unit squares is cut along one of its two diagonals. How many of the unit squares are cut into two pieces? Give a formula in terms of m and n.