Viji Thomas, SUNY Binghamton, Binghamton University

The nonabelian tensor product of finite groups is finite: A homology free proof

R. Brown and J. L. Loday first introduced the non-abelian tensor product $G \otimes H$ for groups G and H in context with an application in homotopy theory. Let G and H be groups which act on each other via automorphisms and which act on themselves via conjugation. The actions of G and H are said to be compatible, if ${}^{^hg}h' = {}^{hgh^{-1}}h'$ and ${}^{^gh}g' = {}^{ghg^{-1}}g'$ for all $g, g' \in G$, $h, h' \in H$. The non-abelian tensor product $G \otimes H$ is defined provided G and H act compatibly. In such a case $G \otimes H$ is the group generated by the symbols $g \otimes h$ with relations $gg' \otimes h = ({}^gg' \otimes {}^gh)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^hg \otimes {}^hh')$, where ${}^gg' = gg'g^{-1}$ and ${}^hh' = hh'h^{-1}$. In their 1987 paper, *Some computations of non-abelian tensor products of groups*, Brown, Johnson and Robertson mention eight open problems. The first problem is phrased as follows: Let G and H be finite groups acting compatibly on each other. Then is it true that $G \otimes H$ is finite? In the same year, G. J. Ellis answered the question in the affirmative using homological methods. Brown, Johnson and Robertson add that no purely algebraic proof is known. In this talk I will present a homology free and purely group theoretic proof that the non-abelian tensor product of two finite groups is finite.

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RJ Dolbin, UC Riverside

Ideals in parabolic subalgebras of simple Lie algebras

We study ad-nilpotent ideals of a parabolic subalgebra of a simple Lie algebra. Any such ideal determines an antichain in a set of positive roots of the simple Lie algebra. We give a necessary and sufficient condition for an antichain to determine an ad-nilpotent ideal of the parabolic. We write down all such antichains for the classical simple Lie algebras and in particular recover the results of D. Peterson.

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Timothy Ridenour, University of California, Riverside

Rigid subsets of weights for simple Lie algebras

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$. Let $wt(V)$ be the set of weights for a finite dimensional $\mathfrak{g}$-module, V. A subset S of $wt(V)$ is said to be rigid if $\sum_{\beta \in S} r_\beta \beta = \sum_{\mu \in wt(V)} m_\mu \mu$ with r_β, $m_\mu \in \mathbb{Z}^+$ for all β, μ implies $\sum_{\beta \in S} r_\beta \leq \sum_{\mu \in wt(V)} m_\mu$ with equality holding if and only if $m_\mu = 0$ for all $\mu \notin S$. I will discuss the classification of rigid subsets for irreducible $\mathfrak{g}$-modules and describe some properties which relate rigid sets to the weight polytope of V.

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Brian Miller,Texas Tech

Differential Galois Theory and the Descent Problem

Let $K \supset k$ be differential fields of characteristic 0. Let M be a differential object defined over K. We say that M descends to N if there exists a differential object N over k such that $M \simeq K \otimes_k N$. More generally, the descent problem consists of finding differential fields ℓ where $K \supset \ell \supset k$ such that M descends to ℓ. We present some background information on differential Galois theory and some expected results in connection with the descent problem.

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Ibrahim Saleh, KSU

Cluster automorphisms

Let $\mathcal{A}_n(S)$ be a cluster algebra over a field K. A cluster automorphism is an element of $Aut._K K(t_1, t_2 \cdots, t_n)$ that leaves the set of all cluster variables χ_S invariant. The group of all such automorphisms is described in terms of the orbits of the symmetric group action on the set of all seeds S.

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Paula Murgel Veloso, University of Sao Paulo, Brazil

Special units, unipotent units and free groups in group algebras

Necessary and sufficient conditions under which the group of units $\mathbb{U}(KG)$ of the group ring KG does not contain a free subgroup of rank 2 are established in [1]. Let G be a finite group. If K is a field of characteristic zero, then $\mathbb{U}(KG)$ does not contain a free subgroup of rank 2 if and only if G is abelian. On the case of a field K of positive characteristic π, $\mathbb{U}(KG)$ does not contain a free subgroup of rank 2 if and only if either G is abelian, or K is algebraic over its prime subfield $GF(\pi)$, or $G/\mathbb{O}_\pi(G)$ is abelian, where $\mathbb{O}_\pi(G)$ denotes the largest normal π-subgroup of G.

Let K be a field of characteristic $\pi \geq 0$ and $\lambda \in K$ be a transcendental element over its prime subfield $GF(\pi)$. Let G be a group. Given a non-trivial bicyclic unit $1 + (1 - a)b\widehat{a} \in \mathbb{U}(KG)$, we consider the unipotent unit $1+\lambda(1-a)b\widehat{a} \in \mathbb{U}(KG)$. In this work [2], we consider the problem of whether the subgroup $\langle(1 + \lambda(1 - a)b\widehat{a}), (1 - \lambda b)\rangle$ of $\mathbb{U}(KG)$ is a free group in two contexts: first, when *char* $K = 0$ and G is a non-abelian finite group; and then when *char* $K > 0$ and G is a non-abelian torsion group.

This is a joint work with Prof. J. Z. Gonçalves.

References:

[1] J. Z. Gonçalves, *Free subgroups of units in group rings*, Canad. Math. Bull., Vol. 27 (1984), No. 3, 309 – 312.

[2] J. Z. Gonçalves, P. M. Veloso, *Special units, unipotent units and free groups in group algebras*, AMS Contemporary Mathematics (to appear), 2009.

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Ines Henriques,University of Nebraska-Lincoln

Koszul modules over short Gorenstein local rings

We identify a class of local rings $(R, \mathfrak{m})$ with $\mathfrak{m}4 = 0$ exhibiting the Koszul like property that $H_R(-t)P^R_M(t) \in \mathbb{Z}[t]$ for all finite R-modules M; where $H_R(t)$ denotes the Hilbert series of R and $P^R_M(t)$ the Poincaré series of M over R. This class includes generic graded Gorenstein algebras of socle degree 3.
We show that minimal free resolutions of finite modules over such rings admit Koszul syzygy modules.

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Qian Lin, MIT

Springer fiber and representations of affine Lie algebra

Recently, there is a categorical version for Deligne-Langlands conjecture, i.e. equivalence between the equivariant coherent sheaves on Steinberg variety and certain perverse sheaves category on affine flag variety. Meanwhile, there is another category equivalence between the perverse sheaves category to the certain representation category of affine Lie algebra. This gives a possibility to replated geometry of springer fiber and representations of affine hecke algebra.

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