Name _____

Finite Applications of Mathematics - MATH 312
Second Exam
October 21, 2005

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ find A^{-1}.

2. In Busytown there are only two industries — coal and electricity. For each \$1 of output the coal industry requires 20 cents input from itself and 20 cents input from the electricity industry. For each \$1 of output the electricity industry requires 30 cents input from the coal industry and 30 cents input from itself.

(a) Give the input–output matrix for this economy, list coal first.

(b) How much should each industry produce to meet an outside demand of \$1000 coal and \$1000 in electricty?

3. A shipping company ships the containers of two companies A and B. The containers from A weigh 40 kgs and are 2 cubic meters. Those from B weigh 40 kgs and are 4 cubic meters. The shipping firm charges company A \$20 for each container shipped and company B \$30 for each container shipped. The shipping company's barges can carry at most 36,000 kgs and cannot hold more than 2000 cubic meters. How many of A's containers and how many of B's containers should the shipping company put into each barge so as to maximize the amount of money they take in?

4. Using the simplex method, find the maximum of $4x + 2y$ subject to the constraints

$$\begin{cases} x + y \leq 20 \\ 3x + 2y \leq 76 \\ x \geq 0, \ y \geq 0. \end{cases}$$

Although it is possible to solve this problem geometrically, you will receive credit only if you use the simplex method.

5. In a group of 50 students it is found that 36 own a cellphone, 31 own a computer, 33 own an iPod, 23 own a cellphone and computer, 24 own a computer and an iPod, 26 own a cellphone and an iPod, and 20 own all 3.

(a) How many own just an iPod?

(b) How many own none of the 3 devices?

6. A group of 4 adults and 3 children is to be photographed.

(a) How many ways can they be arranged in one row?

(b) How many ways can they be arranged with the adults in the back row and the children in the front row?

7. An art museum owns 10 paintings and 8 sculptures. How many possible exhibits can they have which consist of 3 paintings and 4 sculptures?

8. Draw a Venn diagram and shade the portion corresponding to each set.

(a) $A \bigcap B'$

(b) $R \bigcup (S \bigcap T')$

9. Use DeMorgan's laws to simplify each expression.

(a) $(S' \bigcap T')' \bigcap S$

(b) $T \bigcup (S \bigcap T)'$

10. A sandwich shop offers 5 types of bread, 3 types of fillings (chicken, cheese, or ham) and 4 types of spreads (mayonnaise, mustard, ketchup or barbeque sauce.).

(a) How many sandwiches can be created which have bread, a filling and a spread?

(b) Suppose you must have bread and at least either a filling or a spread, but you aren't required to have both a filling and a spread. How many possible sandwiches are there?