FINITE APPLICATIONS OF MATHEMATICS

MATH 312 – EXAM II

OCTOBER 16, 2000 ____________________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

1. A farmer has 100 acres on which to plant oats or corn. Each acre of oats requires \$18 of capital and 2 hours of labor. Each acre of corn requires \$36 of capital and 6 hours of labor. Labor costs are \$8 per hour. The farmer has \$2100 available for capital and \$2400 available for labor. The revenue from each acre of oats is \$55, and the revenue from each acre of corn is \$125.

(9) (a) Determine the objective function for <u>profit</u>. (Be careful, profit is not the same as revenue.)

(9) (b) Write out the linear constraints associated with the problem of maximizing the objective function.

(8) (c) Set up the first simplex tableau for this problem.

(8) (d) Perform the first step in the usual simplex algorithm, to get the 2nd simplex tableau.

2. The shaded region, depicted below, is the feasible set for a two-variable linear programming problem, with objective function $M = ax + by$ to be maximized.

In parts (a) and (b), below, determine at which of the points, A, B, C, D, O this maximum occurs.

(9) (a) (Given $a = 1$, $b = 0$, so $M = x$.)

(9) (b) (Given $a = 3$, $b = 2$, so $M = 3x + 2y$.)

3. Consider the following problem:

"Minimize $3x - y + 2z$ subject to the linear constraints

$$x - y - z \geq -10$$
$$2x + y + z \leq 15$$
$$(x, y, z \geq 0)."$$

(8) (a) Write this problem in matrix form.

(8) (b) Express the dual problem in matrix form.

(8) (c) Express the dual problem in the form of linear constraints and of an objective function to be maximized.

4. Consider the minimization problem:

"Minimize $2x + 3y$ subject to the constraints

$$\begin{aligned} x + y &\geq 10 \\ 2x - y &\geq 12 \\ (x, y &\geq 0)." \end{aligned}$$

(8) (a) Set up the first simplex tableau for this problem, and perform a first step in (there is perhaps that one choice of how to proceed) the simplex algorithm to obtain the next tableau.

(8) (b) Perform a second step.

(8) 5. A certain maximization problem has the following final simplex tableau:

$$\left[\begin{array}{ccccc|c|c} x & y & z & u & v & M & \\ 1 & 3 & 2 & 0 & -2 & 0 & 3 \\ 0 & -1 & 1 & 1 & -1 & 0 & 4 \\ \hline 0 & 2 & 2 & 0 & 7 & 1 & 28 \end{array}\right]$$

What are the values of the variables x, y, and z, and of the slack variables u and v, which yield the maximum value $M = 28$.