Math 312
Exam 1
Spring 1994

Name _____

Be clear and complete. Show all work. No calculators are allowed.

(10) 1. You are given the fact that

$$\begin{bmatrix} 2 & -4 & 6 \\ 4 & -9 & 14 \\ -2 & 6 & -9 \end{bmatrix}^{-1} = \begin{bmatrix} 3/2 & 0 & 1 \\ -4 & 3 & 2 \\ -3 & 2 & 1 \end{bmatrix}.$$

Use this fact to solve the system:

$$\begin{aligned} 2x - 4y + 6z &= 10 \\ 4x - 9y + 14z &= -6 \\ -2x + 6y - 9z &= 11 \end{aligned}$$

(10) 2. (a) Multiply the matrices:

$$\begin{bmatrix} 2 & 1 \\ -2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & -1 & 0 \\ -4 & 3 & -2 & 5 \end{bmatrix} =$$

(10) (b) Find the inverse of $\begin{bmatrix} -2 & 4 \\ 6 & 5 \end{bmatrix}$.

(8) (c) Using determinants decide whether or not the matrix $\begin{bmatrix} 4 & -2 & -5 \\ -2 & 2 & 6 \\ 3 & 0 & -2 \end{bmatrix}$ has an inverse. Give a reason for your answer.

(10) 3. (a) Graph the region of feasibility for the constraints

$$\begin{aligned} 3x - 2y &\leq 12 \\ 2x + y &\leq 5 \\ x &\geq 0 \end{aligned}$$

Find all its vertices.

(6) (b) Minimize $z = 3x + 4y$ subject to the constraints in part (a).

(12) 4. An economy produces three products: widgets, gadgets and fidgets.

Each \$1 of widgets requires \$0 of widgets, \$.60 of gadgets and \$.20 of fidgets.

Each \$1 of gadgets requires \$.30 of widgets, \$.10 of gadgets and \$.50 of fidgets.

Each \$1 of fidgets requires \$.40 of widgets, \$.30 of gadgets and \$.20 of fidgets.

Suppose there is an outside (external) demand of \$1000 of widgets, \$1500 of gadgets and \$2000 of fidgets.

Write the consumption matrix, C. Also, write the demand vector, D. Then write a matrix equation to find the production vector, X, that will meet the demand. DO NOT SOLVE THE EQUATION, JUST WRITE A MATRIX EQUATION FOR X. DO NOT ACTUALLY COMPUTE THE INVERSE OF ANY MATRIX.

$C =$

$D =$

$X =$

(8) 5. (a) Write the initial simplex table for maximizing $z = 5x_1 - 6x_2 + 4x_3$ subject to:

$$\begin{aligned} -x_1 - 2x_2 + x_3 &\leq 4 \\ 2x_1 - x_2 + x_3 &\leq 10 \\ x_1 \geq 0, x_2 \geq 0, x_3 &\geq 0 \end{aligned}$$

(8) (b) Solve the system in part (a) by the **simplex method**. Be careful and complete. Tell which values of x_1, x_2, and x_3 give the maximum z and what that maximum is.

(8) (c) Write the initial simplex table to **minimize** $z = -3x_1 + x_2$ subject to:

$$\begin{aligned} 2x_1 + x_2 &\geq 5 \\ x_2 &\geq 4 \\ x_1 - x_2 &\leq 2 \\ x_1 \geq 0, x_2 &\geq 0. \end{aligned}$$

DON'T SOLVE THE SYSTEM.

(10) 6. Graph the plane $2x + 4y + 6z = 12$. Show the intercepts **and** the traces.