Finite Applications of Math, Midterm Exam. February, 2006

Your Name Here $\longrightarrow\longrightarrow$

1. (10 points) For what numbers x will the following be true ?

$$\begin{pmatrix} x & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ -7 \\ \frac{5}{4} \end{pmatrix} = 0.$$

2. (15 points) A certain system of linear equations has the following augmented matrix.

$$\begin{pmatrix} 2 & 4 & 2 & 4 & 2 & 0 \\ 4 & 6 & 4 & 6 & 4 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & 2 & 1 & 0 \end{pmatrix}$$

Let the names of the variables be x, y, z, u, and v, in that order. Row-reduce the matrix (this involves only a very few steps), and find the solution set of the system.

3. (20 points) In a certain town, in a certain occupied country, it is found that, each year, 25% of the population that were not insurgents become insurgents, and that 85% of the population that already were insurgents remain so. Moreover, this happens year after year.

(a) Write down a system of linear equations which relates the current percentages of the town's population which are insurgents on the one hand, or non-insurgents on the other, to the corresponding percentages for next year. (Be sure to clearly state the meanings that you're assigning to the variables in your system.)

(b) Express your answer in part (a) as a matrix equation.

(c) Use the method of inverse matrices to find a matrix equation which relates the percentages for insurgency and non-insurgency for this year as a function of the corresponding percentages for the preceding year.

4. (40 points) A shipping company is buying new ships. The larger ships cost \$50 million apiece and hold 320 tons of cargo, while the smaller ships cost \$30 million dollars and hold 200 tons of cargo. The company has budgeted \$1080 million (i.e. \$1.08 billion) for the new ships, and wants no more than 30 of them. Due to docking limitations, it wants no more than 15 of the larger ships.

(a) Set up a system of constraints and an objective function for the linear programming problem of maximizing the number of tons of cargo that the company will be able to ship simultaneously.

(b) Sketch (and shade in) the feasible set for this linear programming problem.

(c) How many ships of each kind should the company purchase, in order to maximize its shipping capacity ?

5. (20 points) Consider the linear programming problem: minimize $4x + 2y$ subject to the constraints $x + 2y \geq 4$, $x + 4y \geq 6$, $x \geq 0$, $y \geq 0$.

(a) Write the problem in matrix form.

(b) Write the dual of the problem in matrix form.

(c) Use the simplex method to solve the original problem (by finding the final tableau for the dual problem).