Name _____

Finite Applications of Mathematics - MATH 312
First Exam
September 21, 2005

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. Solve the system using Gaussian elimination.

$$\begin{aligned} x + y - 3z &= 0 \\ 3x - y + z &= 2 \\ 5x + 3y - 5z &= 6 \end{aligned}$$

2. On a warm summer day the temperature x hours (here $x \leq 9$) after sunrise is given by $T = 4x + 70$.

(a) At noon it is $94°$. When was sunrise?

(b) At what time was it $84°$?

(c) What will the temperature be at 3 PM?

3. The demand curve for a certain t-shirt is given by $p = -.02q + 16$. The supply curve is given by $p = .02q + 12$. At what price should the shirt be sold so that supply exactly meets demand and what is the resulting quantity of sales?

4. The cost of shipping a package between two cities is a linear function of the weight of the package. It costs \$5 to ship a 2 kg. package and \$17 to ship a 10 kg. package.

(a) Write an equation which gives the cost of shipping, c, as a function of the weight w.

(b) Interpret the c intercept of this graph.

5. Graph the feasible set for the system of inequalities. Carefully label the feasible set as well as the y intercepts of the lines involved and any corner points of the feasible set.

$$\begin{cases} 2x - y \geq -1 \\ x + y \geq 4 \end{cases}$$

6. Consider the data points $(1, 0)$, $(2, 1)$ and $(3, 4)$. Using the given formulas, find the straight line that provides the best least squares fit to the data.

$$m = \frac{n(x_1y_1 + \cdots + x_ny_n) - (x_1 + \cdots + x_n)(y_1 + \cdots + y_n)}{n(x_1^2 + \cdots + x_n^2) - (x_1 + \cdots + x_n)^2}$$

$$b = \frac{(y_1 + \cdots + y_n) - m(x_1 + \cdots + x_n)}{n}$$

7. A student is decorating her room with posters. There are three types available costing \$4, \$5 and \$10. If she wants to spend exactly \$30 and wants to purchase exactly 5 posters, what are her options?

8. Morgan's diet consists entirely of two foods – hamburgers (H) and fries (F). The matrix M represents the number of units of vitamins A, B, C in each 1 lb. portion size of each food.

$$M = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{H} \\ \text{F} \end{array}$$

Let $L = \begin{array}{c} \begin{array}{cc} H & F \end{array} \\ [2 \ 1] \end{array}$ and $D = \begin{array}{c} \begin{array}{cc} H & F \end{array} \\ [3 \ 2] \end{array}$ represent the number of portions of each food he eats at lunch and dinner respectively. Calculate and interpret the following.

(a) LM

(b) $(L + D)M$

9. In a certain university, each year 10% of the math majors switch to English and 20% of the English majors switch to math. The remaining math majors and English majors do not change their major. (Assume no one new comes to the university and on one ever graduates, they just change majors!)

$$\begin{aligned} \text{Let } x &= \text{ the number of math majors this year,} \\ y &= \text{ the number of English majors this year,} \\ u &= \text{ the number of math majors next year, and} \\ v &= \text{ the number of English majors next year.} \end{aligned}$$

(a) Write a system of equations relating x, y to u and v.

(b) If this year there are 110 math majors and 90 English majors, how many were in each major last year?

10.(a) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$.

(b) Is the point $(2, 3, 1)$ in the feasible set of the system of inequalities below?

$$\begin{cases} x + y + x \geq 0 \\ y - x \geq 1 \\ x \geq 4 \end{cases}$$