FINITE APPLICATIONS OF MATHEMATICS - MATH 312

EXAM I NAME ______________ ______________

SEPTEMBER 18, 2000 ______________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1. Make a diagram for the 'feasible set' for the following system of linear inequalities:

$$\begin{aligned} x + y &\le 10 \\ x + 2y &\le 12 \\ y &\le 4 \\ x \ge 0, \quad y &\ge 0 \end{aligned}$$

(Label the "corners" of the feasible set.)

(15) 2. Use the method of gaussian elimination to find <u>all</u> the solutions to the given system of linear equations.

$$\begin{aligned} x + y + z + w &= 1 \\ 4x + 2y + z + 3y &= 3 \\ 2x - z + w &= 1 \end{aligned}$$

(15) 3. A certain system of linear equations has the matrix form

$$\begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 1 & k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$$

Find the value(s) for k which will guarantee that the system has a solution.

(20) 4.a) Find the inverse of the matrix using the gaussian elimination method.

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

b) Use the answer to part (a) to solve the following system (even though "back-substitution" is easier.)

$$\begin{aligned} x + y - z &= 1 \\ y + z &= 1 \\ z &= 1 \end{aligned}$$

(10) 5. Find the inverse of the matrix $\begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix}$, given that α and γ are non-zero.

(20) 6. A teacher estimates that of the students who pass a test, 85% will pass the next test, while of the students who fail a test, 30% will pass the next test. Let x and y denote the number of students who pass and fail, respectively, a given test. Let u and v be the corresponding numbers for the following test.

Write a matrix equation relating $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} u \\ v \end{pmatrix}$.