Name _____

Finite Applications of Mathematics - MATH 312
Final Exam
December 14, 2005

Show all your work in the space provided under each question. Each problem is worth 10 points.

1. A fair die is tossed 10 times. What is the probability of getting at least 3 sixes?

2. ACME industries makes two products, widgets and thingys, which remarkably only require widgets and thingys to manufacture. To make \$1 of widgets requires 20¢ of widgets and 40¢ of thingys. To make \$1 of thingys requires 40¢ of widgets and 20¢ of thingys.

a. Give the input-output matrix for this manufacturing process – list widgets first.

b. How many widgets and how many thingys should ACME produce to meet an outside demand of \$1000 in widgets and \$4000 in thingys?

3.(a) Five players on a basketball team are to be introduced to the crowd. How many possible orders are there to do this?

(b) A committee consisting of 4 basketball players is to be created with 2 chosen from the men's team and 2 chosen from the women's team. If the men's team has 12 players and the women's team has 10 players, how many possible committees are there?

4. A class has 8 boys and 12 girls.

(a) 4 students are chosen at random. What is the probability that exactly 2 boys and 2 girls are chosen?

(b) What is the probability that no boys are chosen?

5. The reliability of a test for cancer is as follows: Of those with cancer, 95% test positive and 5% test negative. Of those who are free from cancer, 90% test negative and 10% test positive. Assume that .5% of the population has cancer. A person is given the test and tests positive. What is the probability he or she has cancer?

6. Let X denote the random variable defined as the number appearing in one toss of a fair die.

(a) Make a table which gives the probability distribution of X.

(b) Find the mean and variance of X.

7. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

8. A survey of 100 students found that 40 like rap music, 30 like country music, 30 like pop, 5 like rap and country, 4 like country and pop, 3 like rap and pop and no one likes all three types of music.

(a) How many like just rap?

(b) How many don't like any of these three types?

9. The average height for American men is 69 inches with a standard deviation of 3 inches. Assuming the heights are normally distributed, what percentage of American men are 72 inches or taller?

10. Megabank offers 3% interest compounded daily on savings accounts. Across the street, Superbank is offering 3.1% compounded quarterly on savings accounts. Which bank should you choose so as to maximize your return?

11. Ellis-van Creveld syndrome affects one out of 14 Amish people. Using a normal approximation to the binomial distribution, estimate the probability that in a group of 140 Amish at least 8 have the syndrome.

12. The Megamassivegiantfoods Corporation is in the fast food restaurant business and wishes to expand. They run two types of restaurants – Glutton World and Hamburger Heaven. Glutton World restaurants cost \$100,000 to build, require 15 employees and generate an annual profit of \$40,000. Hamburger Heaven restaurants cost \$50,000 to build, require 10 employees and generate an annual profit of \$30,000. The company has \$1,000,000 to invest in building new restaurants and can hire at most 180 new employees. How many of each type of restaurant should they build to maximize future annual profits?

13. Using the simplex method, find the maximum of $9x + y$ subject to the constraints

$$\begin{cases} x + 3y \leq 10 \\ 3x + 4y \leq 6 \\ x \geq 0, \ y \geq 0. \end{cases}$$

Although it is possible to solve this problem geometrically, you will receive credit only if you use the simplex method.

14.(a) A quiz has 5 multiple choice questions, each with 4 possible answers. A student randomly guesses on each question. What is the probability the student gets all 5 correct?

(b) A family has 3 children. Given that at least one is a boy, what is the probability that the other two are girls?

15. You win a lottery prize to be paid out in 20 yearly installments of \$50,000 beginning one year from now. Assuming the Lottery Commission can invest money at 6% interest per year, how much should they set aside today so as to be able to meet their future obligation to you?