FINITE APPLICATIONS OF MATHEMATICS

MATH 312 – FINAL EXAM

DECEMBER 13, 2000 ____________________________

TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK. (150 points)

(10) 1. Let $U = \{\text{all people}\}$, $A = \{\text{Americans}\}$,B=$\{$people under the age of 25$\}$, and $C = \{\text{people; who own a car}\}$.

Describe the following sets, using set-theoretic notation. Also, make a Venn diagram displaying each of these sets (by shading them).

(a) Americans older than 25, who own a car

(b) People under the age of 25, who are not Americans or who own a car

(10) 2. Motors, Inc. sold 325 cars with automatic transmission, 216 with power steering, and 89 with both of these options. How many cars were manufactured with at least one of these options?

(10) 3. A club can elect a president and a vice-president in 380 different ways. How many members does the club have?

(10) 4. An exam contains five multiple choice questions, each having four choices. In how many different ways can the exam be completed?

(21) 5. An experiment consists of tossing a coin ten times and observing the sequence of heads and tails. (So, the <u>order</u> in which the heads and the tails occur is observed.)

(a) How many different outcomes are possible?

(b) How many different outcomes have exactly four heads?

(c) How many different outcomes have at least two tails?

(24) 6. Suppose that a red die and a green die are tossed and the numbers on the uppermost faces observed.

(a) What is the probability that the numbers add up to 4?

(b) What is the probability that the sum of the numbers is not 4?

(c) What is the probability that the sum of the numbers is 176?

(d) What is the probability that the sum of the numbers is less than 13?

(21) 7 An urn contains 4 red balls and 5 white balls. Three balls are chosen at random from the urn.

(a) What is the probability that all three balls are white?

(b) What is the probability that exactly two of the balls are white?

(c) What is the probability that at least two of the balls are red?

(10) 8. Let S be a sample space and let E and F be events associated with S. Suppose that $Pr(E) = .6$, $Pr(F) = .5$, and $Pr(E \cap F) = .2$.

(a) Find $Pr(F|E')$.

(b) Find $Pr(E'|F')$.

(10) 9. 30% of the paperback library books are worn and need replacement, while only 5% of the hardcover books are worn. The library's holding are 90% hardcover and 10% paperback. Draw a tree diagram to illustrate how to find the probability that a book chosen at random from the library is worn and needs replacement.

(10) 10. In problem 9, find the probability that a randomly chosen book is a paperback, given that it is worn and needs replacement. (Use Bayes's Theorem, or any other techniques that works.)

(14) 11. The changes in weather in Costa Rica form (for the purposes of this exam) a regular Markov process. Each day is either rainy or sunny. If it rains one day there is a 90% chance it will be sunny the next day; and if it is sunny one day, there is a 60% chance of rain the next day.

(a) Write down the transition matrix for this Markov process.

(b) In the long run, what is the daily likelihood of sunshine? (If you need more space, use the back of this page for calculations.)